SOLUTIONS

ANSWERS

	VERSION A	VERSION B
1)	14	22
2) (a)	2/15	$-\cos\left(\frac{1}{x^2}\right)(2x)$
(b)	$-\sin\left(\frac{1}{x^2}\right)(2x)$	2/15
3) (a)	4/3	9/2
(b)	$\pi\int_0^2 (2x)^2-(x^2)^2\,dx$,	$\pi\int_0^3 (3x)^2-(x^2)^2\,dx$
4)	$\frac{16}{3}$	$\frac{16}{3}$

MATH 2B – MIDTERM 1

PEYAM RYAN TABRIZIAN

Name: ______________________

Student ID: ______________________

Discussion Section time: (please circle)

8 – 9 AM   11 – 12 PM

Instructions: Welcome to Midterm 1! You have 50 minutes to take this exam, for a total of 100 points. **Do not open the exam until instructed to do so**. This is a closed book and closed notes exam and calculators and/or portable electronic devices such as cell phones are **NOT** allowed and should be turned off during the entirety of the exam. Remember that you are not only graded on your final answer, but also on your work as a whole. Write in complete sentences whenever you can. If you need to continue your work on the back of the page, clearly indicate so, or else your work will be discarded. May your luck be integrable! :)

Academic Honesty Statement: I hereby certify that the exam was taken by the person named and without any form of assistance. I also understand that any form of cheating will be subject to disciplinary consequences, pursuant to the UCI Student Code of Conduct 102.1

Signature: ______________________

1		[redacted]	25
2			25
3			25
4			25
Total		100	

Date: Wednesday, January 31, 2018.

25

1. (~~20~~ *points)* Use the **definition** of the integral (in terms of Riemann sums) to evaluate

$$\int_1^3 (2x+3)\,dx$$

You are allowed to use the following facts:

$$\sum_{i=1}^n 1 = n, \quad \sum_{i=1}^n i = \frac{n(n+1)}{2}, \quad \sum_{i=1}^n i^2 = \frac{n(n+1)(2n+1)}{6}, \quad \sum_{i=1}^n i^3 = \frac{n^2(n+1)^2}{4}$$

PREP WORK

$$\Delta x = \frac{3-1}{N} = \frac{2}{N}$$

$$x_i = 1 + i\Delta x = 1 + \left(\frac{2}{N}\right) i = 1 + \frac{2i}{N}$$

$$\int_1^3 2x-3\,dx = \lim_{N\to\infty} (\Delta x) \sum_{i=1}^N f(x_i)$$

$$= \lim_{N\to\infty} \frac{2}{N} \sum_{i=1}^N 2x_i + 3$$

$$= \lim_{N\to\infty} \frac{2}{N} \sum_{i=1}^N 2\left(1+\frac{2i}{N}\right) + 3$$

$$= \lim_{N\to\infty} \frac{2}{N} \sum_{i=1}^N 2 + \frac{4i}{N} + 3$$

$$= \lim_{N\to\infty} \frac{2}{N} \sum_{i=1}^N 5 + \frac{4i}{N}$$

$$= \lim_{N\to\infty} \frac{2}{N}\left(\sum_{i=1}^N 5\right) + \frac{2}{N}\left(\sum_{i=1}^N \frac{4i}{N}\right) \qquad 10+4 = 14$$

//

$$= \lim_{N\to\infty} \frac{10}{N} \sum_{i=1}^N 1 + \frac{8}{N^2} \sum_{i=1}^N i$$

FACTS

$$= \lim_{N\to\infty} \frac{10}{\cancel{N}} (\cancel{N}) + \frac{8}{N^2} \frac{\cancel{N}(N+1)}{2} = \lim_{N\to\infty} 10 + 4 \underbrace{\frac{N+1}{N}}_{\to 1}$$

25

2. ([redacted] *points total;* ~~*10 points each*~~) Find the following:

(a) (15 PTS)

[redacted]

$$\int_0^1 x^3\sqrt{1-x^2}\,dx$$

1) $u = 1-x^2$, $du = -2x\,dx \Rightarrow x\,dx = -\frac{1}{2}du$

2) $u(0) = 1$, $u(1) = 0$

3) $\text{ANS} = \int_1^0 x^2\sqrt{u}\,x\,dx = \int_1^0 x^2\sqrt{u}\left(-\frac{1}{2}du\right) = \int_0^1 x^2\sqrt{u}\left(\frac{1}{2}du\right)$

(b) The derivative (with respect to x) of

(10 PTS)

$$\left(\int_{x^2}^{3} \sin\left(\tfrac{1}{t}\right) dt\right)'$$

$$= \left(F(3) - F(x^2)\right)'$$

$$= -F'(x^2)\,(2x)$$

$$= -f(x^2)\,(2x) = -\sin\left(\frac{1}{x^2}\right)(2x)$$

~~(c) (Warning: The following function is not odd!)~~

[redacted]

$x^2 = 1-u$

$$\downarrow$$

$$= \int_0^1 (1-u)\sqrt{u}\left(\frac{1}{2}du\right) = \frac{1}{2}\int_0^1 u^{\frac{1}{2}} - u^{\frac{3}{2}}\,du$$

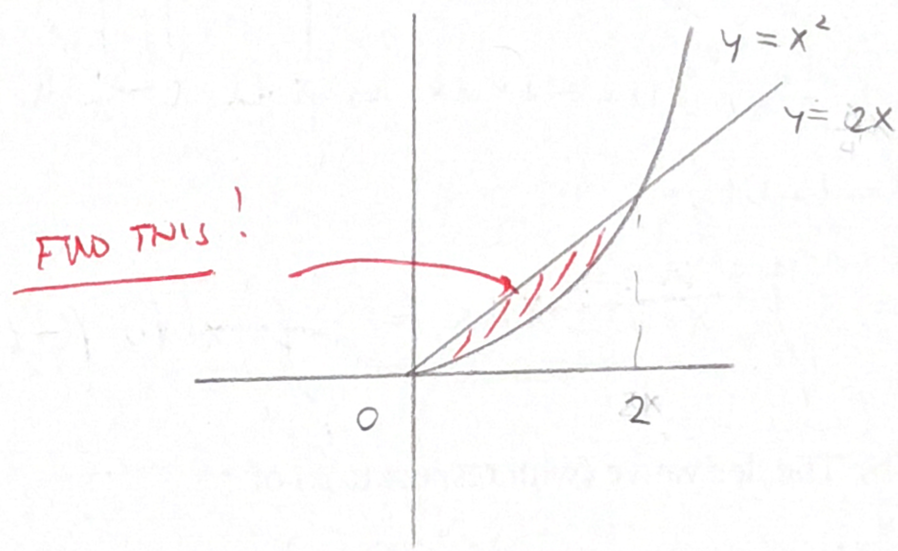
$$= \frac{1}{2}\left[\frac{2}{3}u^{\frac{3}{2}} - \frac{2}{5}u^{\frac{5}{2}}\right]_0^1 = \frac{1}{3}\left(1^{\frac{3}{2}}\right) - \frac{1}{5}(1)^{\frac{5}{2}}$$

$$= \frac{1}{3} - \frac{1}{5} = \frac{5}{15} - \frac{3}{15} = \frac{2}{15}$$

3. ([redacted] 25 *points total)*

(a) ([redacted] 15 *points)* Find the area of the region enclosed by the curves $y = x^2$ and $y =$ [redacted] $2x$

1) PICTURE



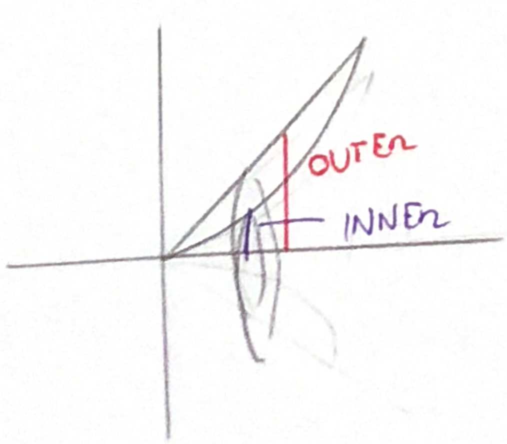
2) POI $\quad x^2 = 2x \iff x^2 - 2x = 0 \iff x(x-2) = 0 \iff x = 0$ or $x = 2$

3) AREA $= \displaystyle\int_0^2 2x - x^2 \, dx$

$= \left[x^2 - \dfrac{x^3}{3} \right]_0^2$

$= 4 - \dfrac{8}{3} = \dfrac{12}{3} - \dfrac{8}{3} = \dfrac{4}{3}$

(b) *(10 points)* Set up, **but do not evaluate**, the volume of the solid obtained by rotating the region in (a) about the [redacted]. X-AXIS



OUTER $= 2x$

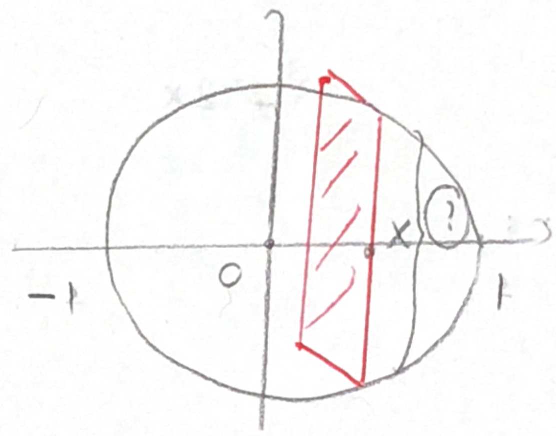
INNER $= x^2$

AXIS $= K = 0$

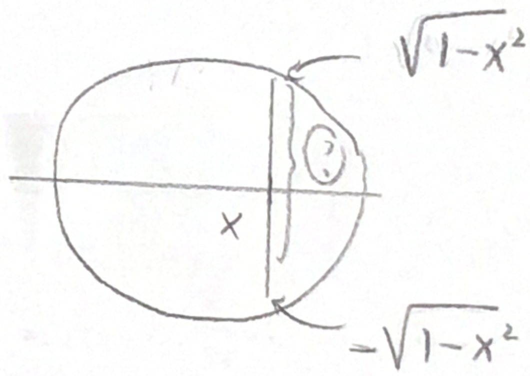
$$\boxed{V = \pi \int_0^2 (2x)^2 - (x^2)^2 \, dx} \quad \left(= \pi \int_0^2 4x^2 - x^4 \, dx \right)$$

4. (~~20~~ 25 *points)* The base of a solid is a circular disk of radius ~~2~~ 1. Cross-sections (perpendicular to the base) are squares. Find the volume of that solid.

1) PICTURE



2) SLICES ARE SQUARES



$A(x) = (?)^2$

BUT $x^2 + y^2 = 1$

$\Rightarrow y^2 = 1 - x^2$

$\Rightarrow y = \pm\sqrt{1-x^2}$

so $? = \sqrt{1-x^2} - \left(-\sqrt{1-x^2}\right) = 2\sqrt{1-x^2}$

3) $V = \int_{-1}^{1} A(x)\,dx = \int_{-1}^{1} \left(2\sqrt{1-x^2}\right)^2 dx = \int_{-1}^{1} \underbrace{4(1-x^2)}_{\text{EVEN}}\,dx$

$= 2\int_0^1 4(1-x^2)\,dx = 8\int_0^1 1 - x^2\,dx$

$= 8\left[x - \frac{x^3}{3}\right]_0^1 = 8\left(1 - \frac{1}{3}\right) = 8\left(\frac{2}{3}\right) = \frac{16}{3}$