

MOCK MIDTERM 1

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Instructions: This is a mock midterm, designed to give you some practice for the actual midterm. It should be similar in length (and in spirit) as the actual midterm.

1		25
2		15
3		30
4		30
Total		100

1. (25 points) Use the **definition** of the integral (in terms of Riemann sums) to evaluate

$$\int_1^4 x^2 dx$$

You are allowed to use the following facts:

$$\sum_{i=1}^n 1 = n, \quad \sum_{i=1}^n i = \frac{n(n+1)}{2}, \quad \sum_{i=1}^n i^2 = \frac{n(n+1)(2n+1)}{6}, \quad \sum_{i=1}^n i^3 = \frac{n^2(n+1)^2}{4}$$

2. (15 points, 5 points each) Find the following:

(a) The antiderivative F of $f(x) = \sec(x)(\sec(x) + \tan(x))$ that satisfies $F(0) = 2$.

(b) $\int_{-3}^3 \sqrt{9 - x^2} dx$

(c) The derivative of $\int_{\frac{1}{x}}^2 \sqrt{1 + e^t} dt$

3. (30 points, 10 points each) Find the following integrals

(a) $\int \frac{1+x}{1+x^2} dx$

Hint: Split the fraction up into two parts

(b) $\int x^3 \sqrt{1+x^2} dx$

(c) $\int_{e^{-4}}^{e^{-1}} \frac{1}{x \ln(x)} dx$

4. (30 points)

(a) (10 points) Find the area of the region enclosed by the curves $y = x$ and $y = \sqrt{x}$. Illustrate with a picture.

(b) (20 points, 5 points each) Find an expression of **but do not evaluate** the volume obtained by rotating the region in (a) about:

- (i) The x -axis
- (ii) The y -axis
- (iii) The line $x = -1$
- (iv) The line $y = 2$