1. **(3 points each)** For each of the following power series, find the radius of convergence and the interval of convergence:

1) \[ \sum \sqrt{n}x^n, \]
2) \[ \sum \frac{1}{n^{\sqrt{n}}}x^n, \]
3) \[ \sum \frac{3^n}{\sqrt{n}}x^{2n+1}. \]

2. For \( n = 0, 1, 2, \ldots \), let \( a_n = \left(\frac{4+2(-1)^n}{3}\right)^n \).

(a) **(4 points)** Find \( \lim \sup |a_n|^{1/n}, \lim \inf |a_n|^{1/n} \), \( \lim \sup |a_{n+1}/a_n| \) and \( \lim \inf |a_{n+1}/a_n| \).

(b) **(3 points)** Do the series \( \sum a_n \) and \( \sum (-1)^na_n \) converge?

(c) **(3 points)** Find the radius of convergence and the interval of convergence of the series \( \sum a_nx^n \).

3. For each \( n \in \mathbb{N} \), let \( f_n(x) = \frac{1}{n} \sin(n^2) \). Each \( f_n \) is differentiable for any \( x \in \mathbb{R} \). Show that:

(a) **(4 points)** \( \lim f_n(x) = 0 \) for any \( x \in \mathbb{R} \).

(b) **(3 points)** \( \lim f'_n(x) \) exists only if \( x \) is a multiple of \( 2\pi \).

4. Show that \( f_n(x) = \frac{n}{nx+1} \), for \( x \in [0, \infty) \) converges pointwise to \( f(x) = \frac{1}{x} \), for \( x \in (0, \infty) \).

**Extra credit:**

1. **(5 points)** Let \( f_n(x) = nx^n \) for \( x \in [0, 1] \) and \( n \in \mathbb{N} \). Show that

(a) \( \lim f_n(x) = 0 \) for \( x \in [0, 1] \).

(b) \( \lim \int_0^1 f_n(x)dx = 1 \).

2. **(5 points)** Let \( f_n(x) = \int_0^x ne^{-ny}dy \), for \( x > 0 \) and \( n \in \mathbb{N} \).

(a) What is the limit \( f(x) = \lim f_n(x) \)?

(b) Do we have \( f(x) = \int_0^x (\lim ne^{-ny})dy \)?