Homework 6

140B

03/04/06
Due date: 03/10/06

1. (5 points each) Compute the upper and lower Darboux sums of the following functions and show that they are integrable on $[0, 1]$. Consider only partitions of the form $P = \{0 = t_0 < t_1 < t_2... < t_n = 1\}$ with $t_k = \frac{k}{n}$.

   (a) $f(x) = 1$.

   (b) $f(x) = x^2 - 1$.

   (c) $f(x) = e^x$.

2. (10 points) Show that the function defined as

   $$f(x) = \begin{cases} 
   x & \text{if } x \text{ is rational}, \\
   0 & \text{if } x \text{ is irrational}
   \end{cases}$$

   is not integrable on $[0, 1]$.

3. (5 points each) Show that if $f(x)$ is integrable on $[a, b]$ then $\alpha f(x)$, for $\alpha \in \mathbb{R}$ is also integrable on $[a, b]$.

Extra Credits:

1. (5 points) Let $f$ be a function integrable on $[a, b]$ and suppose that $g$ is a function such that $f(x) = g(x)$ on $[a, b]$ except for finitely many $x \in [a, b]$. Show that $\int_a^b f(x) = \int_a^b g(x)$.

2. (5 points) Show that if $f$ is integrable on $[a, b]$ then it is also integrable on any interval $[c, d] \subset [a, b]$. 