Homework 1

Math 3D

04/08/05
Due date: 04/15/05

Explain carefully all your answers. Unsupported answers will not receive any credit.

1. (2 points) (from ch. 1.2, ex 4) Solve the equation
   \[ y' + y = te^t \]

2. (2 points) (from ch. 1.2, ex 4) Solve the equation
   \[ (1 + t^2)y' + 4ty = t, \]
   with the condition \( y(1) = \frac{1}{4} \).

3. (4 points) (from ch. 1.2, ex 18) Show that every solution of the equation
   \[ y' + ay = be^{-ct}, \]
   (where \( a \) and \( c \) are positive constant and \( b \) is any real number) goes to 0 as \( t \to +\infty \).

4. (2 points) (from ch. 1.4, ex 4) Solve the equation
   \[ y' = e^{t+y+3} \]

5. (2 points) (from ch. 1.4, ex 6) Solve the equation
   \[ t^2(1 + t^2) + 2yy' = 0, \]
   with the condition \( y(0) = 1 \).

6. (2 points) (from ch. 1.4, ex 8) Solve the equation
   \[ (1 + t^2)^{1/2}y' = ty^3(1 + t^2)^{-1/2}, \]
   with the condition \( y(0) = 1 \).

7. (2 points) (from ch. 1.4, ex 10) Solve the equation
   \[ \cos(y)y' = -\frac{t \sin(y)}{1 + t^2}, \]
   with the condition \( y(1) = \frac{\pi}{2} \).

8. (4 points) (from ch. 1.4, ex 16) Solve the equation
   \[ y' = \frac{t + y}{t - y}. \]