Homework 6

Math 3D

05/20/05
Due date: 05/27/05

Explain carefully all your answers. Unsupported answers will not receive any credit.

1. (2 points) (Not taken from the book) Rewrite the differential equation as a system of first-order differential equation:
   \[ y^{(3)} - 3y = e^t. \]

2. (2 points) (Not taken from the book) Write the system of differential equations in the form \( \dot{x} = Ax \) (\( A \) being a matrix):
   \[
   \begin{align*}
   x_1' &= x_1 + x_2 - x_3 \\
   x_2' &= 3x_1 - x_2 + 4x_3 \\
   x_3' &= -x_1 - x_2
   \end{align*}
   \]

3. (4 points) (from ch. 3.4, ex 4) Find a basis for the solutions of the differential equation:
   \[ \dot{x} = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{pmatrix} x. \]

4. (4 points) (from ch. 3.4, ex 6) Determine whether the given vectors are a basis for the solutions of the differential equations
   \[
   \dot{x} = \begin{pmatrix} 4 & -2 & 2 \\ -1 & 3 & 1 \\ 1 & -1 & 5 \end{pmatrix} x,
   x^1 = \begin{pmatrix} e^{2t} \\ e^{2t} \\ 0 \end{pmatrix},
   x^2 = \begin{pmatrix} 0 \\ e^{4t} \\ e^{4t} \end{pmatrix},
   x^3 = \begin{pmatrix} e^{6t} \\ 0 \\ e^{6t} \end{pmatrix}.
   \]

5. (4 points) (from ch. 3.8, ex 2) Find the general solution of the differential equation:
   \[ \dot{x} = \begin{pmatrix} -2 & 1 \\ -4 & 3 \end{pmatrix} x. \]

6. (4 points) (from ch. 3.8, ex 12) Solve the initial-value problem
   \[
   \dot{x} = \begin{pmatrix} 3 & 1 & -2 \\ -1 & 2 & 1 \\ 4 & 1 & -3 \end{pmatrix} x.
   \]
   with the condition
   \[ x(0) = \begin{pmatrix} 1 \\ 4 \\ -7 \end{pmatrix}. \]