Section 2.3

3: Infinite sequence \( \{a_n\} \) converges to 1. Infinite series \( \Sigma_{n=0}^\infty a_n \) does NOT converge. If \( \lim_{n \to \infty} a_n \) is NOT 0, then \( \Sigma_{n=0}^\infty a_n \) diverges. Check out your class-notes for details.

4(a) converges to 3/13. This is a geometric series with \( q = -3/10 \) and \( \alpha = 3/10 \). Note that \( \alpha \) is the first term with \( n = 0 \).

4(b) converges to 12. In fact you can write this series as \( \Sigma_{n=1}^\infty \frac{3^n}{4^n} \). This is also a geometric series with \( q = 3/4 \) and \( \alpha = 3 \). Note that \( \alpha \) is the first term with \( n = 1 \).

4(c) converges to 2001 + \( \frac{1}{9} \). Again, this series is a geometric series with \( q = 1/10 \) and \( \alpha = 1/10 \).

4(d) converges to 7.5 and has two geometric series. They are \( \Sigma_{n=0}^\infty \left( \frac{2}{3} \right)^n \) and \( \Sigma_{n=0}^\infty \left( \frac{4}{5} \right)^n \). The first one converges to 2.5 and the second one converges to 5.

7 diverges by oscillations.

9(a) diverges since \( \lim_{n \to \infty} a_n \) is NOT 0.

9(b) is the same as \( \Sigma_{n=4}^\infty \left( \frac{1}{4} \right)^n \). Again, this is a geometric series and converges to 1/192, with \( q = 1/4 \) and \( \alpha = 1/256 \). Note that \( \alpha \) is the first term with \( n = 4 \).

9(c) is divergent, check out your class-notes for details.

Section 4.2

1(a) diverges since \( \lim_{n \to \infty} a_n \) is NOT 0.

1(c) converges to \( \frac{3^5}{7 \times 4^7} \). This series looks complicated, but in fact it is a geometric series with \( q = -3/4 \) and \( \alpha = \frac{3^5}{4^3} \). Note that \( \alpha \) is the first term with \( n = 1 \).

1(e) converges. See your class-notes for details.

3 diverges. For this problem, you can write the recursive relation as \( \frac{a_{n+1}}{a_n} = \frac{3n + 10}{n + 1} \), so \( \lim_{n \to \infty} \frac{a_{n+1}}{a_n} = 3 > 1 \). According to the ratio test, this series diverges.

4(a) converges, you can use root test. See your class-notes for details.

4(b) converges, you can use ratio test. See your class-notes for details.

8(a) diverges, since \( \Sigma_{n=1}^\infty \frac{1}{n} \) diverges.

8(d) diverges and in fact it works like \( \Sigma_{n=1}^\infty \frac{1}{n} \), use comparison test and \( \Sigma_{n=1}^\infty \frac{1}{n} \) as a candidate to compare, you know it diverges.

Section 3.2

2 use ratio test, you know that \( |x + 2| < 1 \).

10 use root test, and you know \( |3x - 2| < 1 \).