Noncommutative solitons and intersecting D-branes

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We construct intersecting D-branes as noncommutative solitons in bosonic and type II string theory. "Defect" branes, which are D-branes containing bubbles of the closed string vacuum, play an important role in the construction.

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I. INTRODUCTION

Brane configurations that contain a tachyonic mode are unstable and will decay. The decay modes vary depending on the unstable D-brane configurations involved (see the reviews [1–3]). In bosonic string theory, all D-branes are unstable. The tachyon on the brane may condense to solitonic solutions of the open string tachyon field theory. These solitons have been noted to correspond to lower dimensional unstable D-branes [4,5]. In type II super-strings, there are stable tachyon vortex solutions arising from $D\bar{D}$-brane annihilation [6,7] corresponding to codimension-2 stable Bogomol'nyi-Prasad-Sommerfield (BPS) D-branes.

It was noticed [8,9] that the construction of solitonic D-brane solutions is remarkably simplified in the presence of an infinite nonzero B field. The tachyon field theory becomes noncommutative and, using the operator formalism, the D-brane solutions, now noncommutative (NC) solitons [10], are easily constructed. These NC solitonic D-brane solutions have since been improved to allow for an arbitrary nonzero finite B field [11].

In this paper, we present new decay modes of unstable brane configurations in the presence of a nonzero B field. Our aim is to construct decay modes within the context of the NC operator formalism corresponding to intersecting brane configurations. The intersecting branes we construct all have a nonzero B field in their world volume.

In Sec. II, we carry this out within the context of bosonic string theory, using as a specific example the decay of unstable D25-branes. We find that configurations of two intersecting D$(p-2)$-branes can be constructed only from the decay of two or more Dp-branes. Starting from one Dp-brane, one can obtain perpendicular brane configurations consisting of D$(p-4)$-branes plus defect D$(p-2)$-branes. Defect branes are D-branes containing bubbles of the closed string vacuum. The defect Dp-brane soliton is time independent and asymptotically approaches the solution of the Dp-brane. We conclude the section by constructing nonperpendicularly intersecting D-branes.

In Sec. III, we extend our results to the type II superstrings. Special to the superstring case is that perpendicularly intersecting brane configurations can be constructed from the decay of a single Dp-$\bar{\text{D}}p$ system. In particular, it is possible to obtain a D$(p-2)$-brane perpendicularly intersecting a $\bar{\text{D}}(p-2)$-brane from the decay of a single Dp-$\bar{\text{D}}p$ brane configuration.

In labeling coordinates, we will let x^i denote the noncommutative directions, x^a the commutative directions, and x^μ all the directions.

II. D25-BRANES DECAY MODES

A. Action and conventions

The effective action for the tachyon field ϕ and the U(1) gauge field in the presence of a nonzero B field in Euclidean space is

$$\mathcal{S}=\frac{c}{g_s}\int d^{26}x\left\{V(\phi-1)\sqrt{\det[g+2\pi\alpha'(B+F)]}+\frac{\alpha'}{2}\sqrt{g}f(\phi-1)g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi+\cdots\right\},\tag{2.1}$$

where we have left out higher derivative terms. V and f are written as functions of $\phi-1$. ϕ has been defined such that at the local maximum $\phi=0$, $V(-1)=1$, and at the local minimum $\phi=1$, $V(0)=f(0)=0$. Thus, the tension of a D25-brane with zero B and F fields is $T_{25}=c/g_s$. With a nonzero B, the tension contains an extra factor $T'_{25}=T_{25}\sqrt{\det(g+2\pi\alpha' B)}$.

As shown in [12,13], the presence of a constant nonzero B field can also be incorporated in the effective action by making the space coordinates noncommutative, i.e., $[x^i,x^j]=i\Theta^{ij}$ with ordinary products replaced by $*$ products. In this description, the closed string metric and coupling are replaced by the open string metric G_{ij} and coupling G_s. The gauge symmetry becomes the noncommutative U(1) with the tachyon transforming in the adjoint representation and the noncommutative field strength $\hat{F}$ appearing only in the combination $\hat{F}+\Phi$, where Φ is an additional parameter. Making the choice $\Phi=-B$, the relations between closed and open string variables for a maximum rank B field are given in Euclidean signature by

$$\Theta=\frac{1}{B},$$

$$G=-(2\pi\alpha')^2B\frac{1}{g}B,\tag{2.2}$$

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$$G_s = g_s \det(2\pi\alpha' B g^{-1})^{1/2}.$$

Introducing $C_i = \Theta^{-1}_{ij} x^j + A_i$ and recalling that the derivative is defined in NC theories as $\partial_i f = -i\Theta^{-1}_{ij}[x^j, f]$, we write

$$D_i\phi = \partial_i\phi - i[A_i,\phi] = -i[C_i,\phi],$$

$$\hat{F}_{ij} + \Phi_{ij} = \partial_i A_j - \partial_j A_i - i[A_i, A_j] - B_{ij} = -i[C_i, C_j]. \tag{2.3}$$

Therefore, the Euclidean noncommutative effective action for a maximum rank B field can be written as

$$\mathcal{S} = \frac{c}{G_s}\int d^{26}x \Big\{ V(\phi-1)\sqrt{\det(G_{ij} - 2\pi\alpha' i[C_i, C_j])} - \frac{\alpha'}{2}\sqrt{G} f(\phi-1) G^{ij}[C_i,\phi][C_j,\phi] + \cdots \Big\}, \tag{2.4}$$

with equations of motion

$$\delta\phi: \quad V'(\phi-1)\sqrt{M} - \frac{\alpha'}{2}\sqrt{G}G^{ij}\{f'(\phi-1)[C_i,\phi][C_j,\phi] + 2[f(\phi-1)[C_i,\phi], C_j]\} = 0,$$

$$\delta C_i: \quad -2\pi i[C_j, V(\phi-1)\sqrt{M}(M^{-1})^{ji}] - \sqrt{G}G^{ij}[\phi, f(\phi-1)[C_j,\phi]] = 0, \tag{2.5}$$

where $M_{ij} = G_{ij} - 2\pi\alpha' i[C_i, C_j]$.

Although we will carry out calculations throughout in Minkowski space and with a nonmaximum rank B field, Eq. (2.4) times an overall minus sign is the action we will use. This is appropriate because all solitonic solutions presented in this paper are independent of the commutative coordinates and have vanishing A_a, the commutative components of the gauge field. All solutions are also exact [14,11] in that each satisfies Eq. (2.5) and also $[C_i,\phi]=0$ and $[C_i,[C_j,C_k]]=0$. The last two conditions ensure that higher derivative terms in the action (2.4) do not affect the equations of motion and the energy of the solution.

Working in the NC operator formalism, fields on noncommutative space become operators acting on an auxiliary infinite-dimensional Hilbert space with dependence only on the commutative coordinates. With a nonzero B field in only two directions, the Hilbert space is $\mathcal{H} = L^2(R)$ and we will use as a basis the eigenstates of the simple harmonic oscillator. The projection operator P_n and the shift operator S_n are defined for $n \geqslant 1$ to be

$$P_n = \sum_{m=0}^{n-1} |m\rangle\langle m|, \quad S_n = \sum_{m=0}^{\infty} |m+n\rangle\langle m|, \tag{2.6}$$

satisfying the relations

$$S_{n_1}S_{n_2} = S_{n_1+n_2}, \quad \bar{S}_n S_n = I, \quad S_n \bar{S}_n = I - P_n, \tag{2.7}$$

$$P_n S_n = \bar{S}_n P_n = 0. \tag{2.8}$$

For Θ^{ij} of rank $2p$, the corresponding Hilbert space can be expressed as a tensor product of p copies of $\mathcal{H}$. As an example, for $p=2$, operators act on the Hilbert space $\mathcal{H}\otimes\mathcal{H}$. Projection operators on such a space are likewise obtained by tensor products, for example,

$$P^{\mathrm{I}}_{n_1}P^{\mathrm{II}}_{n_2} \equiv P_{n_1}\otimes P_{n_2} = \sum_{m_1=0}^{n_1-1}\sum_{m_2=0}^{n_2-1} |m_1, m_2\rangle\langle m_1, m_2|. \tag{2.9}$$

Our notation is that the Roman number superscript on the operator will refer to the Hilbert space on which it acts. The superscript is left out when there is no ambiguity. For instance, P and S are taken to be, respectively, any general projection and shift operator acting on the whole auxiliary Hilbert space. Finally, integrals over noncommutative space are expressed in the operator formalism as

$$\int \frac{d^{2p}x}{\sqrt{\det(\Theta^{ij})}} \to (2\pi)^p \mathrm{Tr}. \tag{2.10}$$

B. D21-branes versus D23-brane with tangential *B* field

To start, we discuss two types of exact solution turning on a nonzero B field in four directions. We work in $25+1$ Minkowski space, taking $g_{\mu\nu} = \eta_{\mu\nu} = (-1,+1,\ldots,+1)$ and $B_{23,22} = B_{25,24} = b > 0$.[1] From Eq. (2.2),

$$\theta = \Theta^{22,23} = \Theta^{24,25} = \frac{1}{b},$$

$$G_{\mu\nu} = \mathrm{diag}(-1,1,\ldots,1,b'^2,b'^2,b'^2,b'^2), \tag{2.11}$$

$$G_s = b'^2 g_s,$$

where $b' \equiv 2\pi\alpha' b$.

The soliton solution for N coincident D21-branes is given by

$$\phi = I - P, \quad C_i = S\Theta^{-1}_{ij}x^j\bar{S}, \tag{2.12}$$

where I is the identity operator and P is a rank-N projection operator both acting on $\mathcal{H}\otimes\mathcal{H}$. S is defined as the operator that satisfies $S\bar{S} = I - P$ and $\bar{S}S = I$. This solution is an exact solution by the solution generating technique described in [11]. We can check that this solution gives the right tension.[2] Indeed, with $\sqrt{\det(M_{ij})} = b'^4 P + (b'^4 + b'^2)(I-P)$, the evaluation of the action for this solution gives

[1]We take Θ^{ij} to be skew diagonalized. Unless explicitly stated as in Sec. II E, we will set $B_{23,22} = B_{25,24}$. Our results can easily be extended to the $B_{23,22} \neq B_{25,24}$ case with no effect on our conclusions.

[2]We integrate the action to find the tension. In general, the noncommutative action may differ from the commutative action by total derivative terms [13]. In calculating the tension, these terms do not contribute because for D-brane soliton solutions the tachyon potential vanishes at infinity.

$$\mathcal{S}=-\frac{c}{g_s b'^2}\left(\frac{2\pi}{b^2}\right)^2 b'^4 V(-1)\mathrm{Tr}P\int d^{22}x\sqrt{-g}$$
$$=-NT_{25}(4\pi^2\alpha')^2\int d^{22}x\sqrt{-g}=-NT_{21}\int d^{22}x\sqrt{-g}, \tag{2.13}$$

where we have used the relation $T_{p-n}=(4\pi^2\alpha')^{n/2}T_p$. Notice that the tension is independent of b. This is a consequence of the background independence of the action, which would be manifest if we had used the variables $X^i=\Theta^{ij}C_j$ (see [15,16]).

Now consider the soliton solution given by

$$\phi=(I-P_1)^{\mathrm{I}}I^{\mathrm{II}},$$
$$C_i=(S_1\Theta_{ij}^{-1}x^j\overline{S_1})^{\mathrm{I}}I^{\mathrm{II}},\quad i=24,25, \tag{2.14}$$
$$C_i=I^{\mathrm{I}}(\Theta_{ij}^{-1}x^j)^{\mathrm{II}},\quad i=22,23.$$

The tachyon field solution is exactly that of a single D23-brane solution with $B_{23,22}\neq 0$. Thus, Eq. (2.14) is the soliton solution of a single D23-brane with tangential B field in the $x^{22,23}$ directions. As a check, we take the solution of Eq. (2.14) and evaluate the action to obtain the expected result

$$\mathcal{S}=-\frac{c}{g_s}(4\pi^2\alpha')\sqrt{1+b'^2}\int d^{24}x\sqrt{-g}$$
$$=-T_{23}\sqrt{1+b'^2}\int d^{24}x\sqrt{-g}. \tag{2.15}$$

Here, $\sqrt{\det(M_{ij})}=b'^3\sqrt{1+b'^2}P_1^{\mathrm{I}}I^{\mathrm{II}}+(b'^4+b'^2)(I-P_1)^{\mathrm{I}}I^{\mathrm{II}}$.

Notice that Eq. (2.14) cannot be generated by the solution generating technique of [11]. If instead we had $C_i=(I-P_1)^{\mathrm{I}}(\Theta_{ij}^{-1}x^j)^{\mathrm{II}}$ for $i=22,23$ in Eq. (2.14) with the other fields remaining unchanged, then the solution would be an infinite number of coincident D21-branes as it satisfies the general form of Eq. (2.12). This solution demonstrates the important role the gauge field plays in determining the interpretation of NC solitonic brane solutions. (This issue was also raised in [16].) Note also that such a distinction between an infinite number of D21-branes and a D23-brane with nonzero tangential B field cannot be made in the limit $\alpha' b\to\infty$ since terms with C_i in the action (2.4) are negligible in this limit.

C. Perpendicular brane configurations and defect branes

Since we have turned on $B_{22,23}$ and $B_{24,25}$, it seems plausible that there may exist solitonic configurations where two D23-branes intersect. The simplest configuration would be that of one D23-brane with transverse coordinates $x^{22,23}$ intersecting with another D23-brane with transverse coordinates $x^{24,25}$. Here, each D23-brane contains a rank-2 tangential B field. Important to the NC branes construction is that the tachyon operator ϕ must be of the form $I-P$, where P is a projection operator of finite or infinite rank. This form is required to simplify the evaluation of $V(\phi-I)$ in the action and $V'(\phi-I)$ in the equations of motion. The solution of a single D23-brane in Eq. (2.14) suggests that a likely candidate for a configuration of two perpendicularly intersecting D23-branes which is also an exact solution is

$$\phi=(I-P_1)^{\mathrm{I}}(I-P_1)^{\mathrm{II}},$$
$$C_i=(S_1\Theta_{ij}^{-1}x^j\overline{S_1})^{\mathrm{I}}I^{\mathrm{II}},\quad i=24,25, \tag{2.16}$$
$$C_i=I^{\mathrm{I}}(S_1\Theta_{ij}^{-1}x^j\overline{S_1})^{\mathrm{II}},\quad i=22,23.$$

This gives $I-\phi=P_1^{\mathrm{I}}I^{\mathrm{II}}+I^{\mathrm{I}}P_1^{\mathrm{II}}-P_1^{\mathrm{I}}P_1^{\mathrm{II}}$, indeed a projection operator. However, the presence of the term $-P_1^{\mathrm{I}}P_1^{\mathrm{II}}$ naively suggests that around the origin,[3] where the branes intersect, the configuration might involve an additional D21-brane. To be more concrete, we evaluate the action for this configuration. For the solution of Eq. (2.16), we obtain

$$V(\phi-I)=V(-1)\{P_1^{\mathrm{I}}P_1^{\mathrm{II}}+P_1^{\mathrm{I}}(I-P_1)^{\mathrm{II}}+(I-P_1)^{\mathrm{I}}P_1^{\mathrm{II}}\},$$
$$\sqrt{\det(M_{ij})}=b'^4P_1^{\mathrm{I}}P_1^{\mathrm{II}}+b'^3\sqrt{1+b'^2}[P_1^{\mathrm{I}}(I-P_1)^{\mathrm{II}}+(I-P_1)^{\mathrm{I}}P_1^{\mathrm{II}}]+(b'^4+b'^2)(I-P_1)^{\mathrm{I}}(I-P_1)^{\mathrm{II}}, \tag{2.17}$$

which gives

$$\mathcal{S}=-\frac{c}{g_s}\left\{(4\pi^2\alpha')^2+(4\pi^2\alpha')\sqrt{1+b'^2}\,\frac{2\pi}{b}[\mathrm{Tr}_{\mathcal{H}^{\mathrm{I}}}(I-P_1)^{\mathrm{I}}+\mathrm{Tr}_{\mathcal{H}^{\mathrm{II}}}(I-P_1)^{\mathrm{II}}]\right\}\int d^{22}x\sqrt{-g}$$
$$=-\left\{T_{21}+T_{23}\frac{2\pi\sqrt{1+b'^2}}{b}[\mathrm{Tr}_{\mathcal{H}^{\mathrm{I}}}(I-P_1)^{\mathrm{I}}+\mathrm{Tr}_{\mathcal{H}^{\mathrm{II}}}(I-P_1)^{\mathrm{II}}]\right\}\int d^{22}x\sqrt{-g}$$
$$=-T_{21}\int d^{22}x\sqrt{-g}-2\left(T_{23}\sqrt{1+b'^2}\int d^{24}x\sqrt{-g}-T_{21}\frac{\sqrt{1+b'^2}}{b'}\int d^{22}x\right). \tag{2.18}$$

[3]The functional representation of P_1 under Weyl correspondence is $P_1(x,y)=2e^{-(x^2+y^2)/\theta}$ where $[x,y]=i\theta$.

The action calculation (2.18) clearly shows that the configuration is *not* that of two perpendicularly intersecting D23-branes. From Eq. (2.18), the configuration is, however, naturally separated into three components—a D21-brane situated at the origin plus two perpendicularly oriented *defect* D23-branes. The defect of the brane is apparent from the $\mathrm{Tr}_{\mathcal{H}}(I-P_1)$ factor. If the factor was instead $\mathrm{Tr}_{\mathcal{H}}(I)$, then by the trace-integral correspondence of Eq. (2.10), we would have two D23-branes in addition to a D21-brane. The presence of P_1 (or in general P_n) can be thought of as a "bubble" of the closed string vacuum within the unstable D-brane. This can be seen most clearly in taking $\alpha' b\to\infty$. Because of the open string boundary condition, the D23-brane becomes a continuous distribution of D21-branes in the presence of a rank-2 tangential B field [13]. As observed in [9] and evident from the last equality of Eq. (2.18), a defect D23-brane in this limit is simply a D23-brane with one (or in general n) D21-brane subtracted or decayed away. The decay of these D21-branes into the closed string vacuum forms the bubble.

Indeed, the defect brane is an exact static solution for any nonzero value of b. With only a rank-2 B field turned on, $B_{25,24}=b>0$, a defect D25-brane solution is simply

$$\phi=P_1, \quad C_i=S_1\Theta^{-1}_{ij}x^j\overline{S_1}, \quad i=24,25. \tag{2.19}$$

This solution gives a nonzero field strength $\hat{F}_{24,25}=-bP_1$. With $\sqrt{\det(M_{ij})}=b'^2P_1+b'\sqrt{1+b'^2}(I-P_1)$ and $V(\phi-I)=V(-1)(I-P_1)$, the evaluation of the action gives

$$\begin{aligned}\mathcal{S}&=-T_{25}\frac{2\pi\sqrt{1+b'^2}}{b}\mathrm{Tr}_{\mathcal{H}}(I-P_1)\int d^{24}x\sqrt{-g}\\&=-\left(T_{25}\sqrt{1+b'^2}\int d^{26}x\sqrt{-g}-T_{23}\frac{\sqrt{1+b'^2}}{b'}\int d^{24}x\right).\end{aligned}\tag{2.20}$$

In Eq. (2.19), both the tachyon field and the resulting field strength vanish asymptotically. Thus, far away from the origin, the defect D25-brane has the tension of a D25-brane with tangential B field. In general, the defect Dp-brane soliton is a nonconstant solution with the boundary condition that it asymptotically approaches the value of a Dp-brane instead of the closed string vacuum. The defect brane has a nonconstant tension that varies spatially in the noncommutative directions.[4]

We can intuitively understand the presence of defect branes in the decay of a single D25-brane into a perpendicular brane configuration from the "semiclassical" viewpoint. As in Bohr quantization, we can roughly associate each projection operator with an area of the noncommutative space. A D23-brane would involve covering a sheet of noncommutative plane with projection operators such as $P_1^{\mathrm{I}}I^{\mathrm{II}}$. However, an intersection is an overlap region where there must be two projection operators present. But adding two identical projection operators never results in a projection operator. Thus, the requirement that $\phi-I$ be a projection operator prohibits the construction of two intersecting branes at least as a decay mode of a single D25-brane. One of the intersecting branes must have a defect. Here, in our semiclassical description, a defect brane is a D-brane with a "hole." As for the exact solution of Eq. (2.16) consisting of two perpendicular defect branes, the hole that results from the defect is filled with a D21-brane.

We leave to the reader straightforward generalizations of constructing defect branes with "larger bubbles" and other perpendicular configurations by turning on a higher rank B field.

D. Perpendicularly intersecting D-branes

The above semiclassical argument requires that any intersecting D-brane solutions contain two projection operators covering the same noncommutative region. This can be attained by decaying two D25-branes. Simply put, with a rank-4 B field turned on, two intersecting D23 branes is constructed from two D25 branes by letting each D25 brane decay into a D23 brane but with different transverse directions. The fields now have Chan-Paton indices and are 2×2 operator-valued matrices. The noncommutative action with NC U(2) gauge symmetry now has an additional trace over the U(2) representation and we take it to be

$$\begin{aligned}\mathcal{S}=-\frac{c}{G_s}\int d^{26}x\,\mathrm{Tr}\Big\{&V(\phi-1)\sqrt{-\det(G_{ij}-2\pi\alpha' i[C_i,C_j])}\\&-\frac{\alpha'}{2}\sqrt{G}f(\phi-1)G^{ij}[C_i,\phi][C_j,\phi]+\cdots\Big\},\end{aligned}\tag{2.21}$$

Subtle issues of the non-Abelian Dirac-Born-Infeld (DBI) action (see, for example, [17]) will not affect our discussion below and will be ignored. Using Eq. (2.14), the perpendicularly intersecting D23-brane configuration is given by

$$\phi=\begin{pmatrix}(I-P_1)^{\mathrm{I}}I^{\mathrm{II}} & 0\\ 0 & I^{\mathrm{I}}(I-P_1)^{\mathrm{II}}\end{pmatrix},$$

$$C_i=\begin{pmatrix}(S_1\Theta^{-1}_{ij}x^j\overline{S_1})^{\mathrm{I}}I^{\mathrm{II}} & 0\\ 0 & (\Theta^{-1}_{ij}x^j)^{\mathrm{I}}I^{\mathrm{II}}\end{pmatrix}, \quad i=24,25, \tag{2.22}$$

$$C_i=\begin{pmatrix}I^{\mathrm{I}}(\Theta^{-1}_{ij}x^j)^{\mathrm{II}} & 0\\ 0 & I^{\mathrm{I}}(S_1\Theta^{-1}_{ij}x^j\overline{S_1})^{\mathrm{II}}\end{pmatrix}, \quad i=22,23.$$

Since the fields are all diagonal, one easily find that the solution (2.22) gives the expected tension, two copies of (2.15). As a nontrivial check, one can check for the presence of the ground state tachyonic fluctuation mode existing at the intersection. This mode corresponds to an open string connecting

[4]Note that the calculation in Eq. (2.20) does not give the tension of the defect brane. In Eq. (2.20) and also Eq. (2.18), we have simply integrated the contribution of the "bubble" to the action to demonstrate the difference between defect branes and D-branes.

the two branes with both ends at the origin. Indeed, the ground state tachyonic mode arises from the fluctuation of the off-diagonal mode,

$$\delta\phi=\begin{pmatrix} 0 & \beta|0,0\rangle\langle 0,0| \\ \bar{\beta}|0,0\rangle\langle 0,0| & 0 \end{pmatrix}. \tag{2.23}$$

Details of the quadratic fluctuation of the β mode are given in Appendix A 1. The mass of this mode is found to be

$$m_\beta^2=\frac{1}{\alpha'}\left(\frac{1}{\pi b'}-\frac{\sqrt{1+b'^2}}{b'}\right)$$
$$=\frac{1}{\alpha'}\left[-1+\frac{1}{\pi b'}-\frac{1}{2b'^2}+\mathcal{O}\left(\frac{1}{b'^3}\right)\right] \quad \text{as} \quad b'\to\infty. \tag{2.24}$$

As worked out in Appendix A 2, the expected mass from the first quantization calculation is

$$m^2=\frac{1}{\alpha'}\left[-1+\left(\frac{1}{4}-\frac{1}{\pi^2}\tan^{-2}b'\right)\right]$$
$$=\frac{1}{\alpha'}\left[-1+\frac{1}{\pi b'}-\frac{1}{\pi^2 b'^2}+\mathcal{O}\left(\frac{1}{b'^3}\right)\right]$$
$$\text{as} \quad b'\to\infty. \tag{2.25}$$

Comparing Eq. (2.24) with Eq. (2.25), one sees that we have agreement in the large b' limit to order $1/b'$. This is as expected since to obtain agreement to higher order would require working with higher derivative terms of the tachyon field in the effective action (2.21). The exact form of these higher derivative terms is not needed for constructing the exact solution but is essential for calculating the exact masses of the fluctuation modes.

E. Nonperpendicularly intersecting branes

So far we have constructed only perpendicularly intersecting D-branes. It is also possible in the operator formalism to obtain nonperpendicularly intersecting branes. We will demonstrate this for the case of two intersecting D23-branes. This again requires working with the decay of two D25-branes. The solution we seek is of the diagonal form as in Eq. (2.22), but now with one D23-brane with transverse coordinates $x^{24,25}$ and the other D23-brane rotated relative to the first. Since before the decay the noncommutative factor Θ^{ij} is the same on both branes, our goal is to rotate the D23-brane, keeping Θ^{ij} fixed.

As a rank-2 tensor, Θ^{ij}, with $i,j=22,\ldots,25$, transforms under the SO(4) rotation in the four noncommutative directions as

$$\Theta^{ij}\to R^i_k R^j_l \Theta^{kl}, \tag{2.26}$$

where R^i_j is an element of SO(4). With $R=e^{-i\epsilon^A J_A}$, where J_A are the SO(4) generators in the fundamental representation, the requirement that Θ^{ij} is invariant gives the condition

$$[\epsilon^A J_A,\Theta]=0. \tag{2.27}$$

For $\Theta^{22,23}=\theta$ and $\Theta^{24,25}=\theta'$ with $\theta\neq\theta'$ and other components zero, Eq. (2.27) breaks SO(4)=SU(2)×SU(2) down to U(1)×U(1). The U(1)'s correspond to separate rotations in the x^{22}-x^{23} and x^{24}-x^{25} planes. More interestingly, for $\theta=\theta'$, the unbroken group is SU(2)×U(1). Here, the U(1) generator is proportional to $\Theta=i\theta(\underline{1}\otimes\sigma_2)$ where $\underline{1}$ and σ_A are the 2×2 identity matrix and the Pauli matrices, respectively. The SU(2) generators are then

$$J_1=-\tfrac{1}{2}(\sigma_1\otimes\sigma_2), \quad J_2=\tfrac{1}{2}(\sigma_2\otimes\underline{1}), \quad J_3=-\tfrac{1}{2}(\sigma_3\otimes\sigma_2). \tag{2.28}$$

As an example, we will work out the rotation under the J_2 generator. This corresponds to a rotation by an angle φ in both the x^{22}-x^{24} and x^{23}-x^{25} planes, or

$$[R_2]^i_j=\begin{pmatrix} \cos\varphi & 0 & -\sin\varphi & 0 \\ 0 & \cos\varphi & 0 & -\sin\varphi \\ \sin\varphi & 0 & \cos\varphi & 0 \\ 0 & \sin\varphi & 0 & \cos\varphi \end{pmatrix}. \tag{2.29}$$

The unitary transformation of operators under rotation is determined by $U^\dagger x^i U=R^i_j x^j$. Using $[x^i,x^j]=i\Theta^{ij}$, the unitary operator associated with R_2 is found to be

$$U_2=\exp\left(i\varphi\frac{x^{22}x^{25}-x^{23}x^{24}}{\theta}\right). \tag{2.30}$$

We now have the necessary ingredients to write down the solution of a D23-brane with transverse coordinates $x^{24,25}$ intersecting another D23-brane situated at an angle φ in both the x^{22}-x^{24} and x^{23}-x^{25} planes from the first. The solution is

$$\phi=\begin{pmatrix} (I-P_1)^{\mathrm{I}}I^{\mathrm{II}} & 0 \\ 0 & U_2^\dagger(I-P_1)^{\mathrm{I}}I^{\mathrm{II}}U_2 \end{pmatrix},$$

$$C_i=\begin{pmatrix} C'_i & 0 \\ 0 & [R_2]^j_i U_2^\dagger C'_j U_2 \end{pmatrix}, \tag{2.31}$$

having defined C'_i as the gauge field solution C_i in Eq. (2.14) for the single D23-brane with rank-2 tangential B field. One can check that for $\varphi=\pi/2$ Eq. (2.31) becomes the solution of the perpendicularly intersecting D23-branes of Eq. (2.22). This is most easily done working with the annihilation operators

$$a_1=\frac{x^{24}+ix^{25}}{\sqrt{2\theta}}, \quad a_2=\frac{x^{22}+ix^{23}}{\sqrt{2\theta}} \tag{2.32}$$

and their complex conjugate creation operators. As an aside, this operator basis makes explicit that the unbroken SU(2)×U(1)≃U(2) rotation when $\theta=\theta'$ is the U(2) symmetry group of the two-dimensional isotropic oscillator.

III. SUPERSTRING

In the bosonic theory, we have shown that in the presence of at least a rank-4 tangential B field, a D-brane can decay into perpendicular brane configurations. These brane configurations consist of what we called defect branes in addition to lower dimensional branes. To obtain configurations of intersecting D-branes requires the decay of at least two coincident D-branes.

Our results in the bosonic theory can be extended to type II string theory. For the non-BPS D-branes, the results are similar to those of the bosonic theory with some subtleties due to the tachyon potential having two degenerate minima (see [9,18]). For the D$\overline{\text{D}}$ system, the situation is more complex in that the form of the noncommutative effective action for large field strengths is at present not well understood.[5] To sidestep this issue, we analyze the large B field limit which allows us to drop all derivative and gauge field terms. In this limit, the noncommutative action for a type IIB D9-$\overline{\text{D}}$9 system is simply the tachyon potential,

$$\mathcal{S}=-\frac{c}{G_s}\int d^{10}x\sqrt{-G}\{V(\bar{\phi}\phi-1)+V(\phi\bar{\phi}-1)\} \tag{3.1}$$

where the potential is of a Mexican hat–like shape with local maximum at $V(-1)=1$ and minima at $|\phi|=1$. From string field theory [21], the tachyon solution must satisfy the partial isometry condition $\phi\bar{\phi}\phi=\phi$. This is also a sufficient condition for a solution to satisfy the equation of motion of (3.1) with $V(x)=\Sigma_{n=2}^{\infty}a_n x^n$. A solution corresponding to m D7-branes and n D7-branes is given by $\phi=S_n\bar{S}_m$ [11].[6] Notice that this gives

$$\bar{\phi}\phi-I=-P_m\,,\quad V(\bar{\phi}\phi-I)=V(-1)P_m\,,$$

$$\phi\bar{\phi}-I=-P_n\,,\quad V(\phi\bar{\phi}-I)=V(-1)P_n\,. \tag{3.2}$$

The projection operator is again utilized to simplify calculations. Turning on a rank-4 B field, the arguments from Sec. II C imply that perpendicular configurations with defect branes should also exist. For example, for $\phi=\bar{S}_1^{\mathrm{I}}\bar{S}_1^{\mathrm{II}}$, $V(\bar{\phi}\phi-I)=V(-1)\{P_1^{\mathrm{I}}(I-P_1)^{\mathrm{II}}+(I-P_1)^{\mathrm{I}}P_1^{\mathrm{II}}+P_1^{\mathrm{I}}P_1^{\mathrm{II}}\}$. To be certain, one should find the solution for the gauge fields, which we have neglected in the infinite B field limit, and calculate the tension.

One difference in the superstring case is that there exist solutions of D7-branes perpendicularly intersecting $\overline{\text{D}}$7-branes for the single D9-$\overline{\text{D}}$9 decay. For a D7-brane perpendicularly intersecting a $\overline{\text{D}}$7-brane, the tachyon solution is simply $\phi=S_1^{\mathrm{I}}\bar{S}_1^{\mathrm{II}}$. In the semiclassical language of Sec. II C, the projection operators for the D-branes and $\overline{\text{D}}$-branes exist on separate noncommutative planes because the potential in Eq. (3.1) consists of two terms, one associated with D-branes and the other with $\overline{\text{D}}$-branes. However, to obtain a configuration of two intersecting D7-branes requires the decay of two D9-$\overline{\text{D}}$9 branes.

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APPENDIX A: GROUND STATE TACHYON MASS AT THE INTERSECTION OF TWO PERPENDICULARLY INTERSECTING D23-BRANES WITH TANGENTIAL *B* FIELD

1. Soliton fluctuation

We provide details of the calculation for the mass of the ground state tachyonic fluctuation mode. The variation is performed on the NC U(2) action of Eq. (2.21) with respect to the perpendicularly intersecting brane solution of Eq. (2.22). Finding the masses of all the fluctuation modes is highly nontrivial and will not be discussed here. The ground state fluctuation mode comes from the variation of the tachyon field as given in Eq. (2.23). Explicitly, we have

$$\phi+\delta\phi-I=\begin{pmatrix}-P_1^{\mathrm{I}}I^{\mathrm{II}} & \beta P_1^{\mathrm{I}}P_1^{\mathrm{II}}\\ \bar{\beta}P_1^{\mathrm{I}}P_1^{\mathrm{II}} & -I^{\mathrm{I}}P_1^{\mathrm{II}}\end{pmatrix}, \tag{A1}$$

where β is the complex fluctuation field of interest with dependence on x^a, the 22 commutative coordinates. The potential is assumed to be polynomial and have the form $V(\phi-1)=\Sigma_{n=2}^{\infty}a_n(\phi-1)^n$. This gives $O(\beta\bar{\beta})$

$$V(\phi+\delta\phi-I)=\begin{pmatrix}V(-1)P_1^{\mathrm{I}}I^{\mathrm{II}}+\frac{1}{2}V''(-1)\beta\bar{\beta}P_1^{\mathrm{I}}P_1^{\mathrm{II}} & \cdots\\ \cdots & V(-1)I^{\mathrm{I}}P_1^{\mathrm{II}}+\frac{1}{2}V''(-1)\bar{\beta}\beta P_1^{\mathrm{I}}P_1^{\mathrm{II}}\end{pmatrix}. \tag{A2}$$

[5]Recently, the effective action for the D$\overline{\text{D}}$ system has been calculated in [19,20] using boundary string field theory.

[6]Here we choose the convention that the Ramond-Ramond (RR) charge of a D7 brane is $+1$ and is given by the index of ϕ.

We have left out the off-diagonal elements because they do not contribute to the overall trace of the action. This can be seen in that multiplying Eq. (A2) is the diagonal matrix

$$\sqrt{\det(M_{ij})}=b'^3\sqrt{1+b^2}\begin{pmatrix} P_1^{\rm I}I^{\rm II} & 0 \\ 0 & I^{\rm I}P_1^{\rm II}\end{pmatrix}+(b'^4+b'^2)\times\begin{pmatrix}(I-P_1)^{\rm I}I^{\rm II} & 0\\ 0 & I^{\rm I}(I-P_1)^{\rm II}\end{pmatrix}. \tag{A3}$$

As for varying the tachyon kinetic term, it is convenient to work in the basis of annihilation and creation operators of Eq. (2.32). As is conventional, we denote the corresponding complex coordinates as z_i and $\overline{z_i}$, respectively. The kinetic fluctuation has the form given by

$$\frac{\alpha'}{2}\sqrt{G}f(\phi-I)G^{\mu\nu}D_\mu\delta\phi D_\nu\delta\phi = \frac{\alpha' b'^4}{2}f(\phi-I)\Bigg\{-\frac{b}{b'^2}\sum_{i=1}^{2}([C_{z_i},\delta\phi][C_{\overline{z_i}},\delta\phi] + [C_{\overline{z_i}},\delta\phi][C_{z_i},\delta\phi])+\partial^a\delta\phi\partial_a\delta\phi\Bigg\}, \tag{A4}$$

where we have used Eq. (2.11) and noted that $[C_i,\phi]=0$ for the brane solution (2.22). A straightforward calculation gives

$$\sum_{i=1}^{2}([C_{z_i},\delta\phi][C_{\overline{z_i}},\delta\phi]+[C_{\overline{z_i}},\delta\phi][C_{z_i},\delta\phi]) = -\begin{pmatrix}\beta\overline{\beta}P_1^{\rm I}P_2^{\rm II} & 0\\ 0 & \overline{\beta}\beta P_2^{\rm I}P_1^{\rm II}\end{pmatrix}, \tag{A5}$$

and, moreover,

$$f(\phi-I)=f(-1)\begin{pmatrix}P_1^{\rm I}I^{\rm II} & 0\\ 0 & I^{\rm I}P_1^{\rm II}\end{pmatrix},$$

$$\partial^a\delta\phi\partial_a\delta\phi=\begin{pmatrix}\partial^a\beta\partial_a\overline{\beta}P_1^{\rm I}P_1^{\rm II} & 0\\ 0 & \partial^a\overline{\beta}\partial_a\beta P_1^{\rm I}P_1^{\rm II}\end{pmatrix}. \tag{A6}$$

Using Eqs. (A2)–(A6), the action for β up to $O(\beta\overline{\beta})$ is found to be

$$\mathcal{S}=-\alpha' T_{21}f(-1)\int d^{22}x\Bigg\{\partial^a\overline{\beta}\partial_a\beta+\Bigg(\frac{2b}{b'^2}+\frac{1}{\alpha'}\frac{V''(-1)}{f(-1)}\frac{\sqrt{1+b'^2}}{b'}\Bigg)\overline{\beta}\beta\Bigg\}. \tag{A7}$$

By definition, $V''(-1)/f(-1)=-1$ since $\phi=0$ corresponds to the D25 brane. Therefore, the mass of the ground state tachyon is

$$m_\beta^2=\frac{2b}{b'^2}-\frac{1}{\alpha'}\frac{\sqrt{1+b'^2}}{b'}=\frac{1}{\alpha'}\Bigg(\frac{1}{\pi b'}-\frac{\sqrt{1+b'^2}}{b'}\Bigg), \tag{A8}$$

where in the second equality of Eq. (A8) we have used $b'=2\pi\alpha' b$. It is important to remember that this mass formula has been derived neglecting contributions to the quadratic fluctuations from the higher derivative terms of the tachyon field in the effective action (2.4).

2. Ground state mass from string quantization

For completeness, we work out the ground state mass of an open string stretching between two perpendicularly intersecting branes in the presence of a background B field. The open string is parametrized by τ and σ within the region $-\infty<\tau<\infty$ and $0\leqslant\sigma\leqslant\pi$. We will quantize the open string that stretches from the D23-brane with transverse coordinates $x^{24,25}$ at $\sigma=0$ to the D23-brane with transverse coordinates $x^{22,23}$ at $\sigma=\pi$. For coordinates x^a, where $a=0,\ldots,21$, both open string end points satisfy the Neumann boundary condition $\partial_\sigma X^a=0$. With nonzero B fields in the remaining coordinates, the open string boundary conditions are either Dirichlet (D), $\partial_\tau X^i=0$, or mixed (M), $\eta_{ij}\partial_\sigma X^j+2\pi\alpha' B_{ij}\partial_\tau X^j=0$, with

	$\sigma=0$	$\sigma=\pi$	
X^{22},X^{23}	M	D	(A9)
X^{24},X^{25}	D	M	

As in Sec. II D, we take $B_{23,22}=B_{25,24}=b>0$ and let $b'=2\pi\alpha' b$. The boundary conditions are diagonalized in the linear combinations

$$Z^1=\frac{1}{\sqrt{2}}(X^{22}+iX^{23}),\quad Z^2=\frac{1}{\sqrt{2}}(X^{24}+iX^{25}), \tag{A10}$$

and their complex conjugates. In these coordinates, Eq. (A9) becomes

$$\partial_\sigma Z^1+ib'\partial_\tau Z^1|_{\sigma=0}=0,\quad \partial_\tau Z^1|_{\sigma=\pi}=0,$$

$$\partial_\tau Z^2|_{\sigma=0}=0,\quad \partial_\sigma Z^2+ib'\partial_\tau Z^2|_{\sigma=\pi}=0. \tag{A11}$$

The mode expansion is thus shifted and given by

$$Z^1=i\Big(\frac{\alpha'}{2}\Big)^{1/2}\sum_{n=-\infty}^{\infty}[e^{i\nu_1\pi}e^{-i(n+\nu_1)(\tau+\sigma)} - e^{-i\nu_1\pi}e^{-i(n+\nu_1)(\tau-\sigma)}]\frac{\alpha_{n+\nu_1}}{n+\nu_1},$$

$$Z^2 = i\left(\frac{\alpha'}{2}\right)^{1/2} \sum_{n=-\infty}^{\infty} [e^{-i(n+\nu_2)(\tau+\sigma)} - e^{-i(n+\nu_2)(\tau-\sigma)}] \frac{\alpha_{n+\nu_2}}{n+\nu_2}, \tag{A12}$$

where both ν_1 and ν_2 have range $[0,1)$ and Eq. (A11) requires that $\tan \nu_1 \pi = 1/b'$ and $\tan \nu_2 \pi = -1/b'$. The ground state mass is therefore

$$m^2 = \frac{1}{\alpha'}\left[-1 + \frac{1}{2}\nu_1(1-\nu_1) + \frac{1}{2}\nu_2(1-\nu_2)\right]. \tag{A13}$$

This expression can be simplified by letting $\nu_1 = \frac{1}{2} - \epsilon$ and $\nu_2 = \frac{1}{2} + \epsilon$. We thereby obtain

$$m^2 = \frac{1}{\alpha'}\left[-1 + \left(\frac{1}{4} - \epsilon^2\right)\right], \quad \tan \epsilon\pi = b'. \tag{A14}$$

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