Notes on p-Divisible Groups

March 24, 2006

This is a note for the talk in STAGE in MIT. The content is basically following the paper [T].

1 Preliminaries and Notations

Notation 1.1. Let R be a complete noetherian local ring, $\mathfrak{m}$ its maximal ideal. We keep the assumption that the residue field $k = R/\mathfrak{m}$ is of characteristic $p > 0$. (Indeed, most of the definitions and propositions are true for arbitrary noetherian schemes; however, we will work mostly with the described case later on.)

By a finite group scheme G over R, we mean an affine *commutative* group scheme $G = \operatorname{Spec} A$, where A is a finite R-algebra and it is *free* of rank m as an R-module. m is called the order of the group.

It can be shown that if G has order m, then $G \xrightarrow{\times m} G$ is the trivial map. In other words, $\forall x \in G(S)$, $mx = e$ for any R-scheme S. (This is only known as Deligne theorem when G is commutative. It is still a conjecture for noncommutative case.)

The augmentation ideal I is the kernel of the unity map $\epsilon : A \to R$. The differential of G is given by $\Omega_{G/R} = I/I^2 \otimes_R A$. (Using the map $\psi : G \times G \xrightarrow{\sim} G \times G$, given by $\psi(x, y) = (xy, y)$).

For (commutative) finite group scheme $G = \operatorname{Spec} A$, one can define its *Cartier Dual* $G^\vee = \operatorname{Spec} A^\vee$, where $A^\vee = \operatorname{Hom}_{R-mod}(A, R)$ and its multiplication is given by the dual of the comultiplication $m^* : A \to A \otimes A$. The dual $G^\vee$ has a natural structure of group scheme, whose comultiplication comes from the multiplication of the ring of A, and whose inverse is induced by the inverse of G. The construction of dual group scheme is functorial in G and we have a natural isomorphism $G \cong (G^\vee)^\vee$.

An alternative characterization of $G^\vee$ is given by $G^\vee(S) = \operatorname{Hom}_{S-group}(G \times S, \mathbb{G}_m/S)$ for any R-scheme S.

Example 1.2. The dual of $\mu_{p^n} = \operatorname{Spec} R[T]/(T^{p^n} - 1)$ is $(\mathbb{Z}/p^n\mathbb{Z})_R$.

A sequence $0 \to G' \xrightarrow{i} G \xrightarrow{j} G'' \to 0$ is called *exact* if G' is identified as the kernel of j and j is faithfully flat. If the order of G (resp. G', G'') is m (resp. m', m''), then $m = m'm''$.

Example 1.3.
$$0 \to \mu_n \to \mathbb{G}_m \xrightarrow{\times n} \mathbb{G}_m \to 0$$
$$0 \to (\mathbb{Z}/p\mathbb{Z})_k \to \mathbb{G}_a \xrightarrow{x \mapsto x^p - x} \mathbb{G}_a \to 0 \text{ for } R = k,\ \mathrm{char} k = p > 0$$
$$0 \to \alpha_p = \mathrm{Spec}\, k[T]/(T^p) \to \mathbb{G}_a \xrightarrow{x \mapsto x^p} \mathbb{G}_a \to 0 \text{ for } R = k,\ \mathrm{char} k = p > 0$$

Proposition 1.4. *Any (noncommutative) finite flat group scheme $G = \mathrm{Spec}\, A$ over a henselian local ring A admits a canonical functorial* connected-étale decomposition

$$0 \to G^0 \to G \to G^{\text{ét}} \to 0,$$

where $G^0 = \mathrm{Spec}\, A^0$ is the connected component of G and $G^{\text{ét}} = \mathrm{Spec}\, A^{\text{ét}}$ corresponds to the maximal étale subalgebra of A. (The exact sequence has a canonical splitting if $R = k$ is a perfect field of characteristic $p > 0$.)

There is an equivalence of category between the category of finite étale group schemes over R and the category of finite continuous $\mathrm{Gal}(\bar{k}/k)$-modules.

Definition 1.5. Let p be a prime number and h an integer ≥ 0. A *p-divisible group* G over R of *height* h is an inductive system $G = (G_\nu, i_\nu)$, $\nu \geq 0$, where G_ν is a finite group scheme over R of rank $p^{h\nu}$, and we have an exact sequence

$$0 \longrightarrow G_\nu \xrightarrow{i_\nu} G_{\nu+1} \xrightarrow{\times p^\nu} G_{\nu+1}.$$

A morphism between p-divisible groups is a collection of morphisms $(f_\mu : G_\mu \to H_\mu)$ on each level, compatible with the structure of p-divisible groups.

The following are immediate consequences of definition of the p-divisible groups.

- G_μ can be identified with the kernel of $p^\nu : G_{\mu+\nu} \to G_{\mu+\nu}$.
- The homomorphism $p^\mu : G_{\mu+\nu} \to G_{\mu+\nu}$ factors through G_ν since $G_{\mu+\nu}$ is killed by $p^{\mu+\nu}$.
- We have exact sequence $0 \to G_\mu \to G_{\mu+\nu} \to G_\nu \to 0$, (by rank counting).
- The connected-étale decomposition of G_μ gives rise to a decomposition of p-divisible groups $0 \to G^0 \to G \to G^{\text{ét}} \to 0$.

Example 1.6. $\mathbb{G}_m(p) = (\mu_{p^\nu})$ is a p-divisible group of height 1. Let X be an abelian variety then $X(p) = (X[p^\nu])$ has a natural a structure of p-divisible group of height $2n$, where $\dim X = n$.

Recall that we have "finished" the classification of étale part of a p-divisible group, which is an inverse system of $\mathrm{Gal}(\bar{k}/k)$ modules of order $p^{h\nu}$. Now, we need to give some insight to the structure of the connected part.

Definition 1.7. An *n-dimensional formal Lie group over R* is the formal power series ring $\mathcal{A} = R[\![X_1, \cdots, X_n]\!]$ with a suitable comultiplication structure $m^* : \mathcal{A} \to \mathcal{A}\widehat{\otimes}\mathcal{A} = R[\![Y_1, \cdots, Y_n, Z_1, \cdots, Z_n]\!]$, which is determined by $F(Y, Z) = (f_i(Y, Z))$, where f_i are the images of X_i. We require m^* satisfy the following axioms:

1. $X = F(X, 0) = F(0, X)$,
2. $F(X, F(Y, Z)) = F(F(X, Y), Z)$,
3. $F(X, Y) = F(Y, X)$, (since we consider only commutative groups).

Let $\psi : \mathcal{A} \to \mathcal{A}$ denote the multiplication by p in $\mathcal{A}$. We say $\mathcal{A}$ is *divisible* if multiplication by p is an isogeny (surjective map with finite kernel), or equivalently, $\mathcal{A}$ is a finite free module over $\psi(\mathcal{A})$.

Theorem 1.8. *Let R be a complete noetherian local ring with residue field of characteristic $p > 0$. We have an equivalent of category between the category of divisible commutative formal Lie groups over R and the category of p-divisible groups over R.*

Proof. First, given a divisible Lie group $(\mathcal{A}, m^*)$. We define G_ν be the kernel of the multiplication by p^ν in $\mathcal{A}$. In other words, let $I = (X_1, \cdots, X_n)$ be the augmentation ideal of $\mathcal{A}$, and then $G_\nu = \operatorname{Spec} A_\nu$ for $A_\nu = \mathcal{A}/\psi^\nu(I)\mathcal{A}$. It is easy to verify that $G = (G_\nu)$ is a p-divisible group.

To see that this construction gives a fully faithful embedding of the category of divisible formal Lie groups into the category of p-divisible groups, we need to recover $\mathcal{A}$ from the A_ν's, namely, to prove $\mathcal{A} \cong \varprojlim_\nu A_\nu$. This can be done by carefully arguing that topology given by $\psi^\nu(I)$ is exactly the usual topology on $\mathcal{A}$.

Now, we are left to show that given a p-divisible group $G = (\operatorname{Spec} A_\nu)$, we have an isomorphism $\varprojlim_\nu A_\nu \cong R[\![x_1, \cdots, x_n]\!]$. By the formal Nakayama lemma (Here we use essentially that the LHS is a projective limit of Artin rings), it is suffice to prove this for $R = k$. In the field case, we can use the interaction of F (Frobenius) and V (Verschiebung) but I shall omit the details. $\square$

Therefore, for a p-divisible group G, we have two important invariants: the height h and the dimension n (of its connected part).

Example 1.9. $\mathbb{G}_m(p)$ is a p-divisible groups of height 1 and dimension 1. It corresponds to the formal Lie group whose multiplication is given by $F(X) = Y + Z + YZ$.

For an abelian variety X, $X(p)$ is a p-divisible group of height $2n$ and dimension n. However, $X(p)^0$, the identity connected component of $X(p)$, has height between n and $2n$ depending on the Hasse-invariant.

2 Statement of the Main Theorem

Theorem 2.1. *Let R be an integrally closed noetherian domain, whose fraction field is of characteristic 0. Let G and H be p-divisible groups over R. A homomorphism $f : G \otimes_R K \to H \otimes_R K$ of the generic fibers extends uniquely to a homomorphism $G \to H$.*

We shall first give a tentative proof as follow.

Proof. Since R is integrally closed domain, we have $R = \bigcap R_{\mathfrak{p}}$, where $\mathfrak{p}$ runs through all minimal primes of R. Therefore we are reduced to the case of discrete valuation ring. By completion, we are reduced to the case of complete discrete valuation ring (by faithfully flat descent). If the residue field has characteristic$\neq p$, we are working with étale group scheme and the lifting is trivial (since in this case, the category of finite flat group schemes is equivalent to the category of finite Galois modules). Now, the proof follows from the following two lemmas

Lemma 2.2. *If $f : G \to H$ is a homomorphism such that $f \otimes_R K$ is isomorphism, then f itself is an isomorphism.*

Lemma 2.3. *Let H^* be a p-divisible subgroup of $G \otimes_R K$, then there exists a "Néron model" H of H^*, i.e., a p-divisible subgroup H of G such that $H \otimes_R K$ is H^*. (It seems to me that this H might be unique, but there is a little subtlety.)*

The second lemma is just a technical lemma. To prove it, you just pick the "closure" $\tilde{H}_\nu$ of each H^*_ν in G_ν and it turns out that this is almost what you need. However, this $\tilde{H}_\nu$ is not p-divisible. But if we take $H_\nu = \varprojlim_{\mu} \operatorname{Ker}(p^\nu : \tilde{H}_{\mu+\nu} \to \tilde{H}_{\mu+\nu})$, it is not too difficult to show that H_ν is the desired p-divisible group.

Let us assume the first lemma for a while to see how we can deduce the theorem from the two lemmas. This is a standard technique. Given a map $f : G \otimes_R K \to H \otimes_R K$, we can consider its graph $\Gamma^* \subset (G \times H) \otimes_R K$. By the second lemma, we can find a p-divisible subgroup $\Gamma \subset G \times H$. Since $pr_1 \otimes_R K : \Gamma^* \to G \otimes_R K$ is an isomorphism, by the first lemma, we have $pr_1 : \Gamma \to G$ is an isomorphism. Therefore, we have an extended map $pr_2 \circ pr_1^{-1} : G \to \Gamma \to H$. The uniqueness is trivial.

□

Notation 2.4. From now on, we assume that R is a complete discrete valuation ring with residue field k of characteristic p and fraction field K of characteristic 0.

Before proving the first lemma, we first do some observations and see what it tells us. Somehow, it tells us that a p-divisible group is "determined" by its generic fiber. But it is not quite determined. We can give an example of two p-divisible groups whose generic fibers are isomorphic.

Example 2.5. Let $R = \mathbb{Z}_p[\zeta_p]$. Then $\mathbb{G}_m(p)/R$ and $(\mathbb{Q}_p/\mathbb{Z}_p)/R$ have the same generic fiber. But they are obviously non-isomorphic over R although there are morphisms between them.

Tate observed that the only essential difference appearing here was that one of them is étale whereas the other one is not étale. Therefore, he used the *discriminant* to measure the failure of a p-divisible group to be étale.

Definition 2.6. Let $G = \operatorname{Spec} A$ be a finite free group scheme over R. We then have a standard bilinear form $\operatorname{Tr} : A \otimes A \to R$, It extends to a morphism $\phi : \wedge^{\mathrm{top}} A \otimes \wedge^{\mathrm{top}} A \to R$. The *discriminant* $\operatorname{disc}_{G/R}$ of G is defined to be the ideal generated by the image of ϕ.

Example 2.7. Discriminant relates closely to the relative differential. For L/K finite extension and S integral closure of R in L. Then $\mathrm{disc}_{S/R} = \mathbf{N}_{L/K}(\mathrm{Ann}(\Omega_{S/R}))$.

Proposition 2.8. *Let $G = (G_\nu)$ be a p-divisible group. The discriminant of G_ν is $p^{n\nu p^{h\nu}}$, where $n = \dim(G)$, and $h = \mathrm{ht}(G)$.*

It is not hard to see that to prove the first lemma, it is suffice to show that the discriminants of the two p-divisible groups are equal.

Remark 2.9. Tate's philosophy is to prove the first lemma by showing that the two important invariants: height and dimension, are both determined by the generic fiber of a p-divisible group. It is trivial that one can determine the height by the generic fiber; however, it is entirely not obvious that one can get any information about the dimension via the generic fiber of a p-divisible group. On the contrary, dimension reflexes more information about the special fiber.

I think that this paper might be one of the implications that one can read about the information about the special fiber via considering the generic fiber of a lifting. Surprisingly, the information is expected to be independent of the lifting.

3 Hodge-Tate decomposition of Tate Modules

Notation 3.1. Let R be a complete discrete valuation ring with residue field $k = R/\mathfrak{m}$ of characteristic $p > 0$ and quotient field K of characteristic 0. Let $\bar{K}$ denote the algebraic closure of K. $\mathcal{G} = \mathrm{Gal}(\bar{K}/K)$. The completion of $\bar{K}$ is denoted as $\mathbb{C}$, whose ring of integers is $\mathcal{O}_\mathbb{C}$ with maximal ideal $\mathfrak{m}_{\mathcal{O}_\mathbb{C}}$. We have $\mathbb{C}^\mathcal{G} = K$ and $\mathcal{O}_\mathbb{C}^\mathcal{G} = \mathcal{O}_K$.

Let G be a p-divisible group and G^0, $G^{\text{ét}}$ denotes its connected and étale part respectively. And let $\mathcal{A}^{(0,\text{ét})} = \varprojlim_\nu A_\nu^{(0,\text{ét})}$ be its ring of functions.

Let us go through Tate's approach step by step. First of all, Tate interpreted all the information that one can get from the generic fiber by the following two Galois modules:

$$\Phi(G) = \varinjlim_\nu G_\nu(\bar{K}) \qquad \text{via natural inclusion: } G_\nu \to G_{\nu+1},$$
$$T(G) = \varprojlim_\nu G_\nu(\bar{K}) \quad \text{via “multiplication by } p\text{”: } G_{\nu+1} \to G_\nu.$$

Since $\mathrm{char} K = 0$, $G \otimes K$ is naturally étale. $T(G) \cong \mathbb{Z}_p^h$ and $\Phi(G) \cong (\mathbb{Q}_p/\mathbb{Z}_p)^h$ both with action of $\mathcal{G}$. It's not hard to see we have canonical isomorphisms of $\mathcal{G}$-modules

$$T(G) \cong \mathrm{Hom}(\mathbb{Q}_p/\mathbb{Z}_p, \Phi(G)), \qquad \Phi(G) = T(G) \otimes \mathbb{Q}_p/\mathbb{Z}_p.$$

Since G is étale, we conclude that the information we can get from generic fiber are all contained in $T(G)$ or equivalently, $\Phi(G)$. In order to extract information from the two Galois modules, we need to use some “analytic” method.

Definition 3.2. Define $G(\mathcal{O}_{\mathbb{C}}) \stackrel{\text{def}}{=} \mathrm{Hom}_{\text{cont.}}(\mathcal{A}, \mathbb{C})$ to be $\mathcal{O}_{\mathbb{C}}$-points of G. More precisely, $G(\mathbb{C}) = G(\mathcal{O}_{\mathbb{C}}) \stackrel{\text{def}}{=} \varprojlim_i G(\mathcal{O}_{\mathbb{C}}/\mathfrak{m}^i\mathcal{O}_{\mathbb{C}}) = \varprojlim_i \varinjlim_\nu G_\nu(\mathcal{O}_{\mathbb{C}}/\mathfrak{m}^i\mathcal{O}_{\mathbb{C}})$. (Note that $G(\mathbb{C}) = G(\mathcal{O}_{\mathbb{C}})$)

Note here, we use the continuous version of homomorphism, otherwise $G(\mathbb{C}) = G(\bar{K})$ gives us nothing new. We can easily identify the points in $G(\mathcal{O}_{\mathbb{C}})^0$ with the points in $\mathfrak{m}^n_{\mathcal{O}_{\mathbb{C}}}$, since any such map is determined by the image of X_i of $\mathcal{A} = R[\![X_1, \cdots, X_n]\!]$. Note this identification does NOT preserve the group structure.

Example 3.3. For $G = \mathbb{G}_m(p)$, $G(\mathcal{O}_{\mathbb{C}})$ are the points of units in $\mathcal{O}_{\mathbb{C}}$ whose reductions modulo $\mathfrak{m}_{\mathcal{O}_{\mathbb{C}}}$ are 1. $\Phi(G)$ are roots of unity. For $G = X(p)$, $G(\mathcal{O}_{\mathbb{C}})$ are the points whose reductions modulo $\mathfrak{m}_{\mathcal{O}_{\mathbb{C}}}$ are p^{th} power torsions. $\Phi(G)$ are all p^{th} power torsions.

Proposition 3.4. *The torsion part of* $G(\mathcal{O}_{\mathbb{C}})$ *is* $G(\mathcal{O}_{\mathbb{C}})_{\text{tors}} = \Phi(G)$

Proof. Since the p^ν-torsions of $G(\mathcal{O}_{\mathbb{C}}/\mathfrak{m}^i\mathcal{O}_{\mathbb{C}})$ are exactly $G_\nu(\mathcal{O}_{\mathbb{C}}/\mathfrak{m}^i\mathcal{O}_{\mathbb{C}})$. Therefore, the p^ν-torsions of $G(\mathcal{O}_{\mathbb{C}})$ are $G_\nu(\mathcal{O}_{\mathbb{C}}) = G_\nu(\bar{K})$. The proposition follows by taking direct limit on ν. ∎

To study the analytic structure of $G(\mathcal{O}_{\mathbb{C}})$, we observe that $G(\mathcal{O}_{\mathbb{C}})$ naturally has a structure of analytic p-adic Lie group. So, we have a logarithm map from $G^0(\mathcal{O}_{\mathbb{C}})$ to the tangent space $t_G(\mathcal{O}_{\mathbb{C}}) = \{d : I^0/(I^0)^2 \to \mathbb{C}\}$, where I^0 is the augmentation ideal of $\mathcal{A}^0$. (Note in the formula below, $p^i x$ means multiplication in the formal group.)

$$\log x(f) \stackrel{\text{def}}{=} \lim_{i\to\infty} \left(\frac{f(p^i x) - f(0)}{p^i} \right)$$

One can show that this map is a $\mathbb{Z}_p$-homomorphism. Note also, this map is also well defined on $G(\mathcal{O}_{\mathbb{C}})$ since the étale part is torsion. Moreover, restricted to $G^0(\mathcal{O}_{\mathbb{C}})$, log is a local analytic isomorphism $G(\mathcal{O}_{\mathbb{C}})^0 \xrightarrow{\sim} t_G(\mathbb{C})$ and is surjective. Therefore, the kernel of log can be identified with the torsion part of $G(\mathcal{O}_{\mathbb{C}})$, which is exactly $\Phi(G)$.

Summing up the result above, we have an exact sequence

$$0 \to \Phi(G) \to G(\mathcal{O}_{\mathbb{C}}) \xrightarrow{\log} t_G(\mathbb{C}) \to 0.$$

Now, I will list the final result without proof.

- We have natural identification $G^\vee(\bar{K}) = \mathrm{Hom}(G \times \bar{K}, \mathbb{G}_m(p))$. Hence, we have natural pairing

$$\begin{aligned} T(G^\vee) \times G(\mathcal{O}_{\mathbb{C}}) &\longrightarrow \mathbb{G}_m(p)(\mathcal{O}_{\mathbb{C}}) \cong U, \\ T(G^\vee) \times t_G(\mathcal{O}_{\mathbb{C}}) &\longrightarrow t_{\mathbb{G}_m(p)}(\mathcal{O}_{\mathbb{C}}) \cong \mathbb{C}, \end{aligned}$$

  where U denotes $1 + \mathfrak{m}_{\mathcal{O}_{\mathbb{C}}}$, the units in $\mathcal{O}_{\mathbb{C}}$ whose image in the residue field is 1.

- Therefore, we have a commutative diagram of $\mathcal{G}$-modules

$$\begin{tikzcd}
0 \arrow[r] & \Phi(G) \arrow[r] \arrow[d, "\alpha_0"] & G(\mathcal{O}_{\mathbb{C}}) \arrow[r, "\log"] \arrow[d, "\alpha"] & t_G(\mathbb{C}) \arrow[r] \arrow[d, "d\alpha"] & 0 \\
0 \arrow[r] & \mathrm{Hom}(T(G^\vee), U_{\mathrm{tors}}) \arrow[r] & \mathrm{Hom}(T(G^\vee), U) \arrow[r, "\log"] & \mathrm{Hom}(T(G^\vee), \mathbb{C}) \arrow[r] & 0
\end{tikzcd}$$

  where the log map of second line is induced by the natural log map $\log : U \to \mathbb{C}$.

- The α_0 is bijective (trivial), and α and $d\alpha$ are both *injective*. (This is the most nontrivial part of the proof.)

- Tate module inherits the duality of the group: $T(G) = \mathrm{Hom}_{\mathbb{Z}_p}(T(G^\vee), \mathbb{Z}_p(1))$.

- The map $G(R) \xrightarrow{\alpha_R} \mathrm{Hom}_{\mathcal{G}}(T(G^\vee), U)$ and $t_G(K) \xrightarrow{d\alpha_R} \mathrm{Hom}_{\mathcal{G}}(T(G^\vee), \mathbb{C})$ are bijective.

- Therefore, we proved the claim that $T(G)$ determines the dimension n of G via $n = \dim_K t_G(K) = \dim_K \mathrm{Hom}_{\mathcal{G}}(T(G^\vee), \mathbb{C})$. In other words, **the dimension of a p-divisible group equals to the dimension of the Hodge-Tate weight 1 part.**

- We have a canonical exact sequence

$$0 \to t_{G^\vee} \overset{d\alpha}{\to} \mathrm{Hom}(T(G), \mathbb{C}) \to \mathrm{Hom}_{\mathbb{C}}(t_G(\mathbb{C}), \mathbb{C}(-1)) \to 0,$$

  which splits since we have local class field result showing that $H^1(\mathcal{G}, \mathbb{C}(1)) = 0$. (This is cohomological result is completely nontrivial, and it is one of the main result of this paper.)

- Finally, we get a Hodge-Tate decomposition

$$\mathrm{Hom}(T(G), \mathbb{C}) \cong t_{G^\vee}(\mathbb{C}) \oplus (t_G(\mathbb{C}))^\vee(-1).$$

Corollary 3.5. *We have a fully faithful functor $G \to T(G)$ from the category of group schemes over R to the category of Tate modules (with Galois action). Precisely, we have* $\mathrm{Hom}(G, H) \hookrightarrow \mathrm{Hom}_{\mathcal{G}}(T(G), T(H))$.

Remark 3.6. We actually have an interesting result, namely any information about a p-divisible group G over R is contained in $T(G)$ if G itself exists.

Example 3.7. If $G = X(p)$ for some abelian variety X, then the Hodge-Tate decomposition implies that we have a decomposition of the first étale cohomology.

Remark 3.8. This is a surprising result. Namely, we actually proved that whenever given a p-divisible group over a complete discrete valuation ring, the Hodge-Tate weights of it are exactly 0 and -1. **Moreover, Tate also suggests that the coefficients field of exhibit the decomposition should be $\mathbb{C}$.**

Besides all above, the following result might be one of the motivation of the study of p-divisible group.

Proposition 3.9 ([S]). *Given an abelian variety over k, the lifting of the abelian variety to R is equivalent to the lifting of its p^{th} power torsion.*

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