Math 2B
Spring 2017
Midterm 2
05/17/2017
Time Limit: 50 Minutes

Name (Print): ____________________

Student ID ____________________

This exam contains 10 pages (including this cover page) and 5 problems. Check to see if any pages are missing. Enter all requested information on the top of this page, and put your initials on the top of every page, in case the pages become separated.

You may *not* use your books, notes, or any calculator on this exam.

You are required to show your work on each problem on this exam. The following rules apply:

- **If you use a "theorem" you must indicate this** and explain why the theorem may be applied.
- **Organize your work**, in a reasonably neat and coherent way, in the space provided. Work scattered all over the page without a clear ordering will receive very little credit. Moroever, clearly indicate your final answer for each problem.
- **Mysterious or unsupported answers will not receive full credit**. A correct answer, unsupported by calculations, explanation, or algebraic work will receive no credit; an incorrect answer supported by substantially correct calculations and explanations might still receive partial credit.
- If you need more space, use the back of the pages; clearly indicate when you have done this.

Do not write in the table to the right.

Problem	Points	Score
1	15	
2	10	
3	10	
4	15	
5	5	
Total:	55	

1. Determine if the following statements are true or false. For each case explain your answers.

   (a) (3 points)

$$\lim_{n\to\infty} \frac{1+\cos^2\left(e^{n^2}\right)}{n} = 0$$

   (b) (3 points) For a f continuous in $\mathbb{R}$, we have

$$\int_0^e f(x)dx = \int_0^\pi f(x)dx + \int_\pi^e f(x)dx.$$

   (c) (3 points) If f is continuous in $[0,\infty)$ and $\displaystyle\int_1^\infty f(x)dx$ is convergent then, $\displaystyle\int_0^\infty f(x)dx$ is convergent.

(d) (3 points)

$$\lim_{n\to\infty} \sin(2\pi n) = 0.$$

(e) (3 points) The sequence

$$a_n = \cos\left(\frac{n\pi}{n+1}\right)$$

diverges.

2. Evaluate the following integrals

(a) (5 points)

$$\int \cos^2(x)\sin^3(x)dx \tag{1}$$

(b) (5 points)

$$\int \frac{\ln t}{t^4}dx \tag{2}$$

3. Find if the following integrals converge or not, in the case they converge compute them.

   (a) (5 points)

$$\int_0^2 \frac{1}{\sqrt{4-x^2}}\,dx$$

   (b) (5 points)

$$\int_2^\infty \frac{1}{x\ln x}\,dx$$

4. We aim to compute the integral

$$\int \frac{\sin(x)}{\sin(x) + \cos(x) + 1} dx,$$

to do so we will use the substitution

$$u = \tan\left(\frac{x}{2}\right).$$

This question is divided in several parts, you can the results of the parts even if you did not solve them.

(a) (3 points) Show, starting from the trigonometric identity $\sin(x) = 2\sin(x/2)\cos(x/2)$, that:

$$\sin(x) = \frac{2u}{u^2+1}$$

(b) (3 points) Show, starting from the trigonometric identity $\cos(x) = \cos^2(x/2) - \sin^2(x/2)$, that:

$$\cos(x) = \frac{1 - u^2}{u^2 + 1}$$

(c) (2 points) Show that:

$$dx = \frac{2du}{u^2+1}$$

(d) (2 points) Using the substitution provided above, show that

$$\int \frac{\sin(x)}{\sin(x)+\cos(x)+1}dx = \int \frac{2u}{(u+1)(u^2+1)}du.$$

(e) (5 points) Using all the steps above, compute

$$\int \frac{\sin(x)}{\sin(x)+\cos(x)+1}dx.$$

5. (5 points) We define

$$I_n = \int (x+a)^n \sqrt{x+b}\, dx, \qquad a, b, x > 0.$$

Show using integration by parts that I_n satisfies the follow recursion formula:

$$I_n = \frac{2}{3+2n}(x+a)^n (x+b)^{3/2} - \frac{2n(b-a)}{3+2n} I_{n-1}.$$

(**Hint**: you may want to use that $(x+b)^{3/2} = (x+b)\sqrt{x+b} = [(x+a) + (b-a)]\sqrt{x+b}$.)