MATH 2B: SAMPLE MIDTERM #1

- This exam consists of 5 questions and 85 total points.
- Read the directions for each problem carefully and answer all parts of each problem.
- Please show all work needed to arrive at your solutions (unless instructed otherwise). Label graphs and define any notation used. Cross out incorrect scratch-work.
- No calculators or other forms of assistance are allowed. Do not check your cell phones during the exam.
- Clearly indicate your final answer to each problem.

1. (15 points)

   a. Estimate the area under the graph of $f(x) = x^2 + x$ from $x = 0$ to $x = 3$ using 3 approximating rectangles and left endpoints.

   b. Estimate the area under the graph of $f(x) = x - 1$ from $x = 0$ to $x = 6$ using 3 rectangles and midpoints.

   c. Find an expression for the area under the graph of $f(x) = x^2 + x$ from $x = 2$ to $x = 5$ as a limit of Riemann sums. (You do not need to evaluate the limit.)

2. (15 points) Evaluate each of the following indefinite integrals.

a. $\int x\sqrt{3x^2 - 1}\, dx$

b. $\displaystyle\int \frac{1 - \sin^2(x)}{\cos x}\, dx$

c. $\int \sin(7\theta + 5)\, d\theta$

3. (15 points)

a. Find the average value of the function $f(x) = \tan^3(x)\sec^2(x)$ on the interval $[0, \frac{\pi}{4}]$.

b. A particle moves along a line so that its velocity at time t is $v(t) = |2 - t|$. Find the displacement of the particle during the time period $0 \leq t \leq 3$.

4. (20 points)

a. Complete the blanks in the following statement of the Fundamental Theorem of Calculus.

Fundamental Theorem of Calculus:
Suppose f is continuous on $[a, b]$. If $g(x) = \int_a^x f(t)\,dt$, then $g'(x) =$ ____________ and $\int_a^b f(x)\,dx =$ __________________, where F is any antiderivative of f.

b. Use the Fundamental Theorem of Calculus to evaluate the following.

i. $\dfrac{d}{dy}\displaystyle\int_2^y \frac{\sin(t)}{t^2+3}\,dt$

ii. $\dfrac{d}{dx}\displaystyle\int_x^{x^4} \sqrt{t}\,dt$

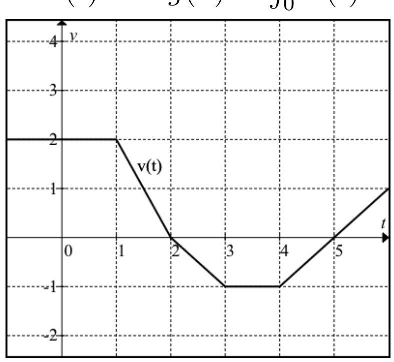
c. Answer each of the following questions. No work or explanation is needed.

i. If $f(t)$ is measured in dollars per year and t in years, what are the units of $\int_0^{10} f(t)\,dt$?

ii. True/False: All continuous functions have derivatives.

iii. True/False: All continuous functions have antiderivatives.

iv. Below is the graph of a function $v(t)$. Let $g(x) = \int_0^x v(t)\,dt$.



Find each of the following:

$g(0) =$ _________, $g(2) =$ _________, $g'(1) =$ _________, $g'(4) =$ _________

5. (20 points) Let S be the region bounded by $y = x^3$ and $y = \sqrt{x}$.

a. Find the area of the region S.

b. i. Find the volume of the solid obtained by revolving the region S about the x-axis.

ii. Set up an integral to find the volume obtained by revolving S about the y-axis. (You do not need to evaluate the integral.)

iii. Set up an integral to find the volume obtained by revolving S about the line $y = 5$. (You do not need to evaluate the integral.)