This sample 2J final exam is meant for practice only. Length and content of the actual final may vary, at the discretion of the instructor.

Try to complete the test in 1 hour and 50 minutes, without consulting any book or notes, and without using a calculator.
### PROBLEM 1 (True/False Questions)

<table>
<thead>
<tr>
<th>Statement</th>
<th>True</th>
<th>False</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}}$ converges.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\sum_{n=1}^{\infty} \frac{1}{n}$ diverges.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>If $\sum_{n=1}^{\infty}</td>
<td>a_n</td>
<td>$ is divergent, then $\sum_{n=1}^{\infty} a_n$ is divergent.</td>
</tr>
<tr>
<td>If $\lim_{n \to \infty} a_n = 0$, then $\sum_{n=1}^{\infty} a_n$ is convergent.</td>
<td>true</td>
<td>false</td>
</tr>
<tr>
<td>$\sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^n = \frac{7}{9}$.</td>
<td>true</td>
<td>false</td>
</tr>
<tr>
<td>$\sum_{n=0}^{\infty} \pi^n = \frac{1}{1-\pi}$.</td>
<td>true</td>
<td>false</td>
</tr>
<tr>
<td>The ratio test can be used to determine whether $\sum_{n=1}^{\infty} \frac{1}{\pi^n}$ converges.</td>
<td>true</td>
<td>false</td>
</tr>
</tbody>
</table>
PROBLEM 2
Determine whether the sequence is convergent or divergent. If it is convergent, find its limit.

<table>
<thead>
<tr>
<th>$a_n$</th>
<th>converges to:</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{\sqrt{n}}{e^n}$</td>
<td>diverges</td>
</tr>
<tr>
<td>$\frac{(-1)^n}{n!}$</td>
<td>diverges</td>
</tr>
<tr>
<td>$(-1)^n \left(3 + \frac{2}{n}\right)$</td>
<td>diverges</td>
</tr>
<tr>
<td>$\frac{\sin(n)}{n^4 + 4}$</td>
<td>diverges</td>
</tr>
</tbody>
</table>
PROBLEM 3

a Find the limit of the sequence

\[ a_n = \arctan n. \]

b Find the limit of the sequence

\[ a_n = \arcsin \left( \frac{3n}{3n + 8} \right). \]

c Find the limit of the sequence

\[ a_n = n \sin \left( \frac{1}{n} \right). \]
PROBLEM 4

Test the following series for convergence or divergence. Motivate your answer, and name the test you use for convergence or divergence.

\[
a \sum_{n=1}^{\infty} \left(3 - \frac{3}{n}\right)^n
\]

\[
b \sum_{n=0}^{\infty} \frac{n^2+5}{n+3n}
\]

\[
c \sum_{n=0}^{\infty} \frac{(-3)^{n+1}}{4^n}
\]
PROBLEM 5

Test following series for convergence or divergence. Motivate your answer, and name the test you use for convergence or divergence.

a \( \sum_{n=2}^{\infty} (-1)^n \frac{\sqrt{n}}{\ln n} \)

b \( \sum_{n=2}^{\infty} (-1)^n \frac{\ln n}{\sqrt{n}} \)

c \( \sum_{n=2}^{\infty} (-1)^n \frac{\ln n}{n^2 \sqrt{n}} \)
PROBLEM 6

For all \( n \geq 0 \), let \( s_n \) be the \( n^{th} \) partial sum of the series \( \sum_{k=0}^{\infty} \frac{1}{2^k} \):

\[
    s_n = \sum_{k=0}^{n} \frac{1}{2^k} = 1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \cdots + \frac{1}{2^n}.
\]

Determine whether the sequence \( \{s_n\} \) converges or diverges.

If it converges, find its sum.
a) Express the function 
\[ \frac{x}{1 - 8x^3} \]
as the sum of a power series centered at \( a = 0 \).

b) Evaluate the indefinite integral 
\[ \int \frac{x}{1 - 8x^3} \, dx \]
as a power series.

*Hint: Use term by term integration.*
PROBLEM 7

Consider the series
\[ \sum_{n=0}^{\infty} \frac{n^3(x - 1)^n}{3^n}. \]

a) Find the radius of convergence.

b) Find the interval of convergence.

*Do not forget to test the end points!*
PROBLEM 8

Find the Taylor series for

\[ f(x) = \sqrt{x} \]

centered at \( a = 1 \).
PROBLEM 9

Find the approximation of

\[ f(x) = 2x^4 + 3x^3 - 2x + 4 \]

by a Taylor polynomial of degree 3 at \( a = 0 \), and estimate the maximum possible error in the approximation when \(-1 \leq x \leq 1\).
PROBLEM 10

a For what values of $x$ does the series $\sum_{n=0}^{\infty} (\ln x)^n$ converge?

*Hint: Think of it as a geometric series.*

b For all $x$ such that the series $\sum_{n=0}^{\infty} (\ln x)^n$ converges, find the sum of the series.
ADDITIONAL PROBLEMS FOR PRACTICE
PROBLEM A

Suppose that \( \{b_n\} \) is a sequence converging to 3, and that \( b_1 = b_2 = 5 \).

Find the sum of the telescoping series \( \sum_{n=1}^{\infty} (b_n - b_{n+2}) \).
PROBLEM B

a Use the ratio test to prove that the series \( \sum_{n=0}^{\infty} \frac{n^n}{(2n)!} \) is convergent.

b What can you deduce about the limit \( \lim_{n \to \infty} \frac{n^n}{(2n)!} \)?