MATH CIRCLE LOGIC PROBLEMS

NAM TRANG

A	B	A $\wedge$ B	A$\vee$ B	if A then B	$\neg$ A
T	T	T	T	T	F
T	F	F	T	F	F
F	T	F	T	T	T
F	F	F	T	T	T

Typically, a statement one wants to prove is of the form: if A then B. Two often-used approaches to show if A then B is true are:

(i) Direct proof: we assume A is true. We try to show B is true.

(ii) Proof by contradiction: we assume A is true and we also assume B is FALSE. We then try to obtain a contradiction from this assumption.

Example. Prove (both by a direct proof and by a proof by contradiction) that for all integers n, if n^2 is odd, then n is odd.

PROBLEMS

1) The four-digit number 4365 has an interesting property. If we write it backwards, and add the original number to the reversed number, we get a string of 9s: 4365 + 5634 = 9999. a) Show that 2097 and 4185 each have the same property.

b) Find two more four-digit numbers that have this property.

c) Find a six-digit number that has this property (that is the sum of it and the number obtained from it by writing it backwards is 999999).

d) Find a three-digit number that has this property.

2) (Lewis Carroll puzzle) Suppose:

1. A says B lies;
2. B says C lies;
3. C says A and B lie.

This set of exercises are taken from Camp Logic: A Week of Logic Games and Activities for Young People by Mark Saul and Sian Zelbo

Determine who lies and who tells he truth. Use both the direct method of exhausting all posibilities (you may want to draw a table that lists all these possibilities) and the method of proof by contradiction.

3) If exactly one of the following is true, which is it?

(a) Puppies are cuter than kittens.

(b) Puppies are cuter than bunnies and kittens.

(c) Puppies are by far the cutest animals.

The next 4 problems concern the pigeonhole principles.

Pigeonhole Principles. if more than n pigeons are placed into n pigeonholes, some pigeonhole must contain more than one pigeon.

4) Ian has a bowling party for his birthday and invites fifteen friends. Show that at least two of the friends must have knocked over the same number of pins on their first turn.

5) Kira's sock drawer has twelve blue socks, thirteen red, and seven green. She reaches in without looking. How many socks must she grab to make sure that she has a matching pair? How many must she grab to be sure that she has three matching pairs?

6) Alek has a bowling party for his birthday and invites fifteen friends. If everyone in the room has at least one friend at the party, then show that two people must have the same number of friends in attendance.

7) A huge deck of cards numbered 1 through 2014 is shuffled. Michael removes cards from the deck, one by one. What is the largest number of cards he must remove to ensure that two of the cards have a difference that is a multiple of 5?

8) The lattice below consists of five columns of five points each, all equally spaced. Lines are drawn connecting pairs of lattice points, but never two in the same row or column. (An example is shown.) How many lines must be drawn to ensure that two of the lines are parallel?

The last two are miscellaneous logic problems.

9) Two mathematicians, Albert and Isaac, chat. Isaac says he has three children who all have the same birthday (but who weren't necessarily born in the same year). Albert asks their ages. Isaac replies, "The product of the ages of my children is 72." Albert points out that this is not enough information to determine their ages. Isaac responds with another clue – he tells Albert the sum of the ages of his children. But Albert again points out that there is not enough information. Finally Isaac says, "My youngest child is named Galileo." At last, Albert correctly determines the ages of Isaac's children. What are the ages?

10) A clock is observed. The hour hand is exactly at the minute mark, and the minute hand is six minutes ahead of it. Later, the clock is observed again. This time, the hour hand is exactly on a different minute mark, and the minute hand is seven minutes ahead of it. How much time elapsed between the first and second observations?