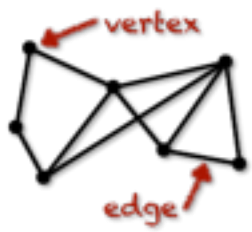
UCI Math Circle **September 28, 2013**

Problem Solving

By Cynthia Northrup

1. Graph Theory
2. The Game of Nim
3. The Calendar Game
4. Operating a Security System
5. Planets
6. Pie and Pawns
7. Games of Stones
8. The Island of Thrice

Problem 1.

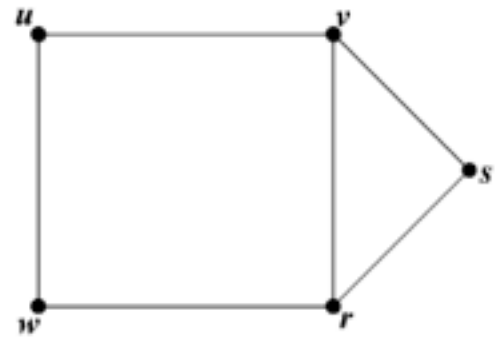


A **graph** is a diagram consisting of vertices which are connected by edges.

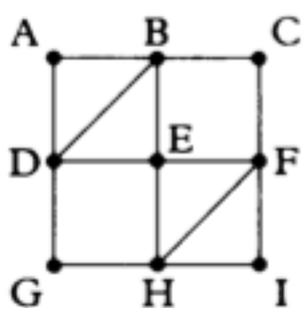
The **degree** of a vertex is the umber of edges which are connected to it. For example, in the graph below we can see that vertex u has degree 2 and vertex r has degree 3.

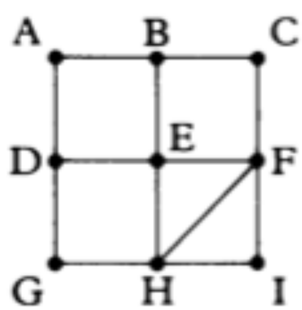
An **Eulerian path** drives along the edges of the graph and uses every edge exactly once.

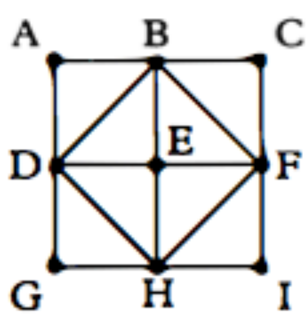
For example, rwuvsrv is an Eulerian path for the graph on the right, but uvsrwu is not (because we never used the edge which connects v and r).

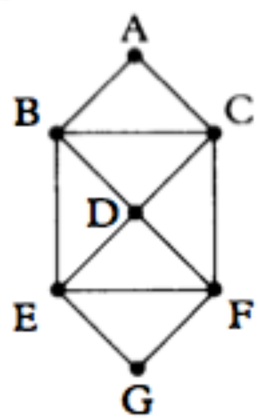


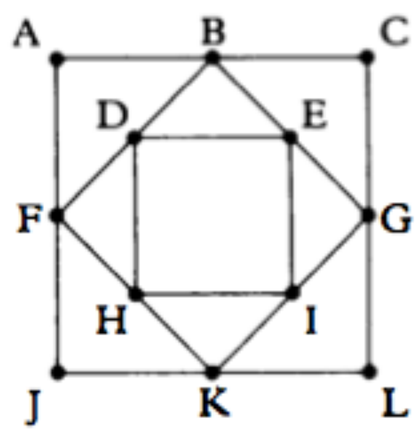
For each of the following examples: determine the degree of each vertex and find all Eulerian paths (if there are any).











Can you see any patterns regarding when a graph must have or not have an Eulerian path?

Problem 2. The Game of Nim

The game of Nim is played as follows: It begins with three piles, which each contain some number of matchsticks. The players alternate turns. Each player, in turn, may select as many sticks (up to the limit of the pile) as desired from any one pile in which there remain any sticks. He or she may not select from more than one pile on any one turn. The player who removes the final stick from the final pile is the winner.

(a) One version of the game begins with piles of 3, 5, and 7 sticks respectively. In this version of the game, is it best to go first or second? What strategy should you follow?

(b) Generalize to the game beginning with three piles containing p, q, and r sticks respectively.

Problem 3. The Calendar Game

Consider the following game with two players, A and B.

A is allowed to select any date of the year (other than December 31). B may then select any date later in the same month or the same day of any later month. For example, if A selects June 16, then B may choose any later date in June or the 16th of any month from July through December. Using the same rule with regard to the date that B has selected, A must select a new date, and so on. The winner is the player who arrives at December 3l.

(a) What date should A select to begin with in order to ensure the fastest possible win ?

(b) What if A must start with a date in January?

Problem 4. Operating a Security System

In order to prevent tampering by unauthorized individuals, a row of switches at a defense installation is wired so that, unless the following rules are followed in manipulating the switches, an alarm will be activated:

1 . The switch on the right may be turned on or off at will.

2. Any other switch may be turned on or off only if the switch to its immediate right is on and all other switches to its right are off.

What is the smallest number of moves in which such a row of switches, which are all on, may be turned off without activating the alarm if:

(a) There are three switches in the row?

(b) There are four switches in the row?

(c) There are five switches in the row?

(d) There are six switches in the row?

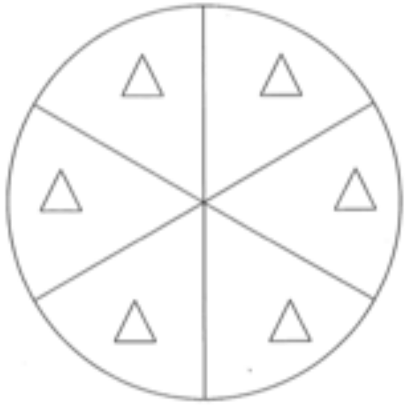
(e) There are an odd number of switches in the row?

(f) There are an even number of switches in the row?

Problem 5.

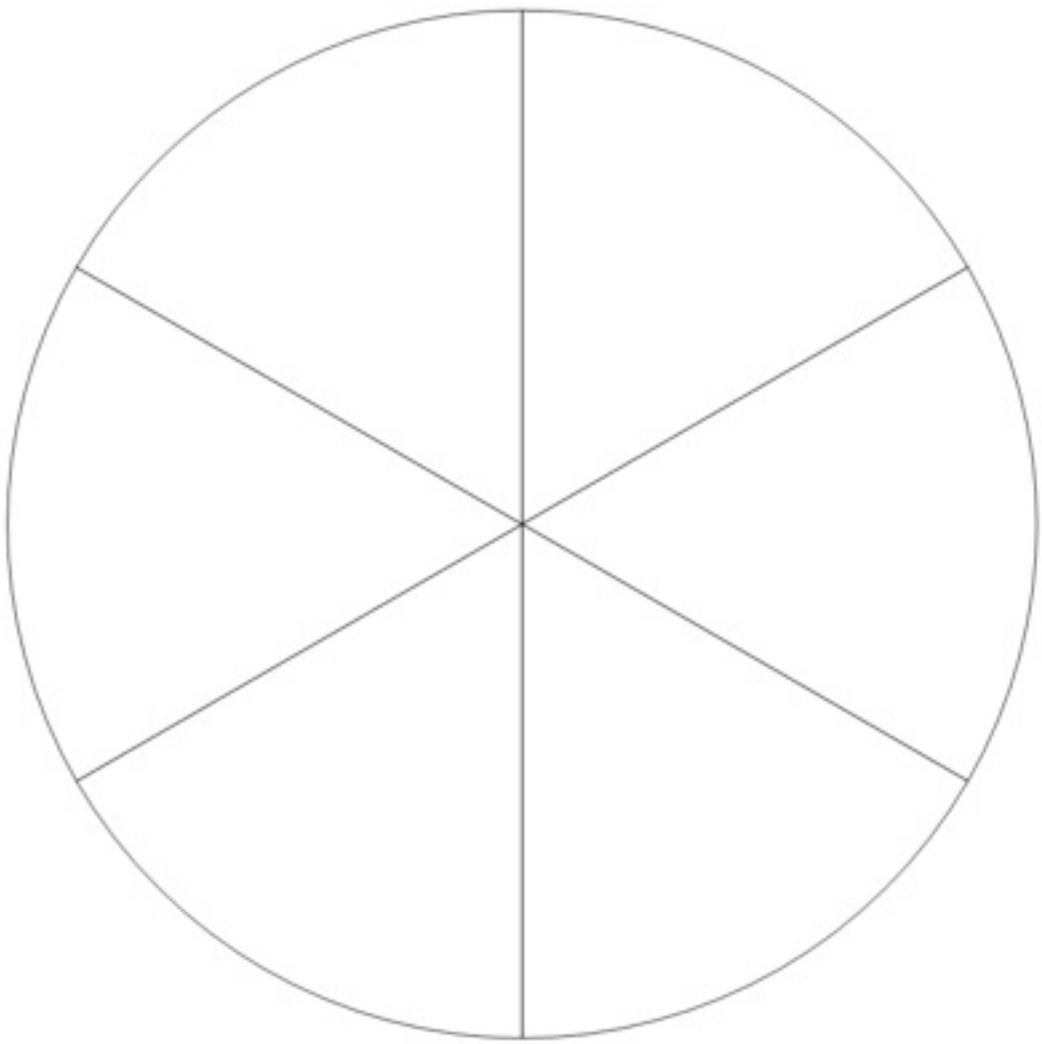
Travel Between the Planets

Cosmic liaisons are established among the eight planets of our solar system, and the dwarf planet Pluto. Rockets travel along the following routes: Earth-Mercury, Pluto-Venus, Earth-Pluto, Pluto-Mercury, Mercury-Venus, Uranus-Neptune, Neptune-Saturn, Saturn-Jupiter, Jupiter-Mars, and Mars-Uranus. Can a traveler get from Earth to Mars?



Problem 6.

A circle is divided into 6 sectors, and a pawn stands in each of them. At each turn you must move two pawns. They may each only move into a sector bordering their original sector. Is it possible to gather all pawns in one sector when following these rules?



Problem 7. Game of Stones

There are three piles of stones: one with 10 stones, one with 15 stones, and one with 20 stones. At each turn, a player can choose one of the piles and divide it into two smaller piles. The loser is the player who cannot do this.

a. Who will win, player 1 or player 2?
b. How will they win?

Problem 8. The Island of Thrice

A group of islands are connected by bridges in such a way that one can walk from any island to any other. A tourist walked around every island, crossing each bridge exactly once. He visited the island of Thrice three times. How many bridges are there to Thrice, if

(a) The tourist neither started nor ended on Thrice?
(b) The tourist started on Thrice, but didn't end there?
(c) The tourist started and ended on Thrice?

From "Problem Solving Through Recreational Mathematics" book:

1. (Graph Theory) Parts of chapter 6 put together
2. (Nim) p.214 #7.3
3. (Calendar) p.257 #7.25
4. (Switches) p.305 #8.2

From "Russian Math Circles" book:

5. (Planets) p.39 Chapter 5 #1
6. (Pie, Pawns) p.124 Chapter 12 #2
7. (Stones) p.58 #2
8. (Island of Thrice) p.49 Chapter 5 #21