

Preference Ballots & Preference Schedules

A preference ballot is used to track everyone's preferences in a situation in order to determine how they will vote. For each person, their preferences are listed from most preferred to least preferred. For example, if Lucas, Nicole, Paul, and Rita are all thinking about primary colors, their preferences could be expressed on a preference ballot:

	Lucas	Nicole	Paul	Rita
1	Red	Red	Blue	Yellow
2	Blue	Yellow	Yellow	Red
3	Yellow	Blue	Red	Blue

In this case, Lucas likes Red the best, Blue the second best, and Yellow the least. If asked to vote on a favorite primary color, we can expect him to vote for Red. If Red were out of contention, Lucas would instead vote for his second-favorite color, Blue. (What would everyone else vote for?)

If we're looking at a lot of people, it's obviously inconvenient to list everyone individually. We can instead note the number of people who have a particular preference, in what's called a *preference schedule*. In the example below, five people have the same preferences as Lucas did, six would agree with Nicole, seven line up with Paul, and five would vote just like Rita. Because the preferences Blue, Red, Yellow and Yellow, Blue, Red are not listed, we know that no one has those preferences.

	5	6	7	5
1	Red	Red	Blue	Yellow
2	Blue	Yellow	Yellow	Red
3	Yellow	Blue	Red	Blue

Warmup

Find five friends and fill out a preference ballot and a preference schedule for the three primary colors. Be sure to list everyone by name on the ballot and by numbers on the schedule!

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Now that you're comfortable with the basic methods, go find a station. There are tables on the back of this form you can use to help solve the problems.

Preference Forms

These tables may be useful for scratch work.

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Plurality Voting

In a *plurality* voting system, the option that gets the most votes wins. Let's look at a plurality system for determining the temperature on the thermostat. Everyone will have an ideal temperature between 40 °F and 90 °F, but the thermostat will only have a limited number of settings. Each person will prefer those settings in order from closest to their ideal to farthest from their ideal. So, for example, if the two settings are 60 °F and 80 °F, the preference ballot is

	40 °F - 70 °F	70 °F - 90 °F
1	60 °F	80 °F
2	80 °F	60 °F

- The thermostat has four settings: 40 °F, 50 °F, 66 °F, and 78 °F.

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- (Easy) Fill out the preference ballot.
 - (Easy) Assuming every temperature between 40 °F and 90 °F is equally preferred, what percentage of the people prefer each option on the preference ballot?
 - (Easy) Assuming everyone votes for their first preference, what temperature will the thermostat be set to?
 - (Easy) What percentage of people got their least-favorite choice?
 - (Medium) If you were trying to make as many people as happy as possible, what do you think is the best setting?
- Let's see what happens now that everyone knows how the election will go.
 - (Medium) Think about those who got their least-favorite option. How would you recommend they vote the next time it's time to change the thermostat? *Hint: Anyone who voted for a second-placed option probably shouldn't change their vote.*
 - (Hard) After the second selection, are there any people who could get a better outcome from changing their vote? What should they change it to? How about after the third selection?
 - (Hard) In the end, what percentage of the people are still voting for their favorite thermostat setting?
 - (Hard) In every situation where the thermostat has only four settings, do you think that two of the settings would still end up getting ignored?
 - (Hard) How about if people could change the setting they voted for by a small amount after each round? (e.g. 50 °F to 54 °F)

Tournament Voting

In a *single elimination tournament* voting system, all of the options are paired off and everyone votes for their favorites in each pairing. The winners are then paired off and the process repeats as necessary until there's only one option left. Let's start by looking at a tournament voting system with only four options for the best superhero: the Incredible Hulk, Iron Man, Captain America, and Thor.

1. Collect an odd number of people (at least 5) and start with Hulk vs. Thor and Iron Man vs. Captain America. For each pair, everyone should vote on who is the cooler superhero. Once you have the two winners, vote on who the cooler of those two is.
 - (a) (Easy) How many different ways are there to set up the initial pairings?
 - (b) (Easy) Assuming everyone agrees on who the coolest is before you start the voting, will that character win? If everyone agrees on the least cool character, is that character guaranteed to lose?
 - (c) (Medium) Do you think it's possible to get a different outcome if you started with different pairings? Run the vote again for each of the other starting pairings to see if something changes.
 - (d) (Medium) If you ran a separate vote where everyone simply voted for their first choice and found a majority winner (a character who got over half the votes), will that character win the tournament?
 - (e) (Hard) How about a plurality winner (a character who got more votes than any other) who didn't get a majority? Construct a preference ballot with either five or seven voters that guarantees that the plurality winner won't win the tournament.
 - (f) (Hard) Construct a preference ballot with either five or seven voters that guarantees that someone who is no one's favorite wins the tournament.
2. Let's think about larger tournaments.
 - (a) (Easy) You can obviously run such a tournament with either two or four options, but not with three (at least not without an option skipping through a round). For what numbers of options will there be no skips (commonly called byes)?
 - (b) (Medium) Assuming the group as a whole can rank its collective preferences for four options, what's the lowest-ranked option that can get to the final round? How about for eight options?
 - (c) (Hard) Determine a rule that works for every number in 2(a) to find the lowest-ranked option that can get to the final round.

Runoff Voting

In a runoff voting system, if no one option gets a majority (more than half the votes), low-scoring options are removed and the vote is run again. Let's look at a runoff system where *only the lowest-scoring option* is removed each time.

- It's time to choose what sort of food to get for lunch at the family reunion. The preference schedule is

	2	6	10	1	2	6
1	Hot Dogs	Burgers	Tacos	Chicken	Salad	Pizza
2	Salad	Salad	Hot Dogs	Salad	Tacos	Chicken
3	Pizza	Chicken	Burgers	Burgers	Burgers	Hot Dogs
4	Burgers	Pizza	Pizza	Hot Dogs	Chicken	Tacos
5	Chicken	Tacos	Salad	Tacos	Pizza	Burgers
6	Tacos	Hot Dogs	Chicken	Pizza	Hot Dogs	Salad

- (Easy) What are the vote totals after the first elimination?
 - (Easy) Which option wins?
 - (Medium) Sometimes it's possible to fast-forward through the early rounds by eliminating multiple options at once. Looking only at the totals from the first round, find two more options that had no chance at winning and thus could have been eliminated as part of the first round.
 - (Medium) Devise a rule to determine when multiple eliminations can be resolved at once.
- Let's now look at the three options that had a fighting chance.

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- (Medium) Fill out a new preference schedule that only takes those options into account.
- (Medium) How many people got their least-favorite of the three options in the end?
- (Hard) Is there any way for these people to get a better option by changing the way they vote in each round? How? *Hint: It might be best for only some of them to change their votes.*
- (Hard) Can they get a better option even if they're restricted to only changing votes between rounds if their preferred candidate is eliminated (i.e. if they must vote according to a particular preference list)? If it's possible, find the preference list they should use for voting. *Hint: Once again, it might be necessary for only some of them to use a new preference list.*

