UCI Math Circle
"Infinity"

WARM UP

There are eight piles of objects located around the room. For each pair of piles, determine which pile has more objects.

Rules: 1. You cannot use any numbers to justify your answers.
2. You cannot compare more than two piles at a time.

	Chopsticks	Cup Lids	Origami	Pasta	Pencils	Pens	Sharpeners	Toothpicks
Chopsticks								
Cup Lids								
Origami								
Pasta								
Pencils								
Pens								
Sharpeners								
Toothpicks								

Questions:

Without using numbers, how can you tell that two sets are the same size?

Without using numbers, how can you tell that two sets are different sizes?

If two sets are not the same size, how can you tell which set is bigger?

Extra question:

Based on the information you collected in the chart, can you order the piles from biggest to smallest?

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EXCERSICES

For each problem, compare the sizes of the sets by pairing the objects in one set with objects in the other set.

1. {chairs in the room} vs. {desks in the room}

2. {chairs in the room} vs. {natural numbers}

3. {desks in the room} vs. {natural numbers}

4. {odd integers} vs. {even integers}

5. {natural numbers} vs {integers}

6. {natural numbers} vs {natural numbers bigger than 5}

7. {natural numbers} vs. {non-negative integers}

8. {integers} vs. {even naturals}

9. {natural numbers} vs. {divisors of 60}

10. {natural numbers} vs. {integers}

UCI Math Circle
"Infinity, Part 2"

WARM UP / REVIEW

Definition: A set is a collection of items.

Examples: The chairs in this classroom form a set
The colors of the rainbow for a set
The collection of all positive integers, which we denote by N, forms a set

Definition: We call an item in a set an element of the set.

Examples: The color blue is an element of the set {colors of the rainbow}
The number 6 is an element of N
The number 5.5 is not an element of N

For each pair of sets, determine which set is larger by pairing the elements of one set with elements of the other set

1. {chairs in the room} vs. {desks in the room}

2. {positive multiples of 3} vs. {positive multiples of 16}

3. {integers} vs. {odd naturals}

Definition: Let A be a set. A subset of A is a set that can be created by removing any number of elements of A.

Examples: {1, 2, 3, 4, 5, 6 } is a subset of **N**
{2, 4, 6, 8, 10, 12, 14, … } is a subset of **N**
{-1, 1, 3, 5, 7, 9, 11, 13, 15, … } is NOT a subset of **N**

Definition: Let A and B be sets. A one-to-one correspondence is a map from A to B that pairs elements of A with elements of B, so that every element is included in exactly one pair.

Definition: Let A be a set. The size of a set A is called its cardinality.
– If you can find a one-to-one correspondence between A and the set {1, 2, …, n} , then A has cardinality *n*
– If you can find a one-to-one correspondence between A and **N**, then A is *countable* (or *infinitely countable*)

For each set, determine its cardinality by finding a one-to-one correspondence with **N** or a subset of **N**

4. {days in one week}

5. {integers larger than -7}

6. {odd integers}

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Determine the rules for finding the cardinality of sets:

If A is a finite set and you remove finitely many elements, the cardinality can become
bigger than / smaller than / the same as the cardinality of A

If B is a finite set and you add finitely many elements, the cardinality can become
bigger than / smaller than / the same as the cardinality of B

If C is an infinite set and you remove finitely many items, the cardinality can become
bigger than / smaller than / the same as the cardinality of C

If D is an infinite set and you add finitely many items, the cardinality can become
bigger than / smaller than / the same as the cardinality of D

If E is an infinite set and you remove infinitely many items, the cardinality can become
bigger than / smaller than / the same as the cardinality of E

If F is an infinite set and you add infinitely many items, the cardinality can become
bigger than / smaller than / the same as the cardinality of F

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Exercises

1. Is **Z** countable?

2. Is **N** x **Z** countable?

3. Is **N** x **Z** x **Z** countable?

Challenge Questions

4. Is **Q** countable?

5. Is **R** countable?