Italia

The Beautiful Land

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Introduction

In this chapter you will join Alessia, Bruno and Marco through a magical and beautiful tour of the country of Italy. You will visit several cities and learn about beautiful cathedrals and monuments, fantastic food and different activities such as soccer. This is a trip not to be missed!
1 The Colosseum

Marco, Alessia and Bruno start their tour of Italy in the beautiful Rome, the capital and heart of Italy, heading to the Colosseum in a warm sunny day.

The Colosseum has a total of 70 zones, numbered from 0 to 69 as the picture shows. A group of doves and crows fly to the Colosseum, landing on some of these zones.

Marco notices the following:
- There is a dove every 5 zones, starting from the second one (zones 2, 7, 12 and so on).
- There is a crow every 4 zones, starting from the second one (zones 2, 6, 10 and so on).

**Doves and Crows**

**a** How many zones have a dove?

**b** How many zones have a crow?

Answer:
Find the largest number of consecutive zones having a combined total of exactly 5 doves.

Which zones have both birds?

How many zones have neither bird?

A Larger Colosseum

Suppose that the Colosseum has 410 zones instead of 70. Complete the following diagram, indicating the number of zones with only doves, only crows, both birds and no birds. Note that the numbers should add to 410.
Marco, Alessia and Bruno are hungry; knowing that Pizza is very tasty, they decide to share a pizza. Each friend has different tastes, however.

Our friends have five choices for the toppings:

Basil (B), cheese (C), mushrooms (M), olives (O) and tomato (T).

- Alessia wants mushrooms and basil.
- Marco wants mushrooms or olives, but no tomato.
- Bruno wants cheese and insists that a good pizza should have at most 3 toppings.

**Pleasing everyone**

Which toppings should be chosen to make all three friends happy, according to the above?

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Did you know...

In mathematics, when we use **OR** we allow the possibility for both things to happen. For example, Marco wants mushrooms or olives, so he would be OK if a pizza has both mushrooms and olives.
Pizza and fractions

Our friends take too long to make up their mind, so the waiter decides to order for them. He comes with the following pizza:

What fraction of the pizza does each friend like?

<table>
<thead>
<tr>
<th></th>
<th>Alessia</th>
<th>Marco</th>
<th>Bruno</th>
</tr>
</thead>
<tbody>
<tr>
<td>O</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>M</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>T</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>C</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>B</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>T</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>C</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>B</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>O</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>M</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>O</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>T</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
3 Calcio Practice

Happily fed, our friends embrace the Italian culture and go see the Azzurri, the National Soccer Team, which is practicing for the World Cup...

The starting eleven players of the Azzurri wear jerseys with numbers from 1 to 11. There is a substitute player wearing 0. We want to split the Azzurri into two teams of 6 players each to play a practice game. We do so in a way that the sum of the T-shirt numbers of the first team is equal to the sum of the T-shirt numbers of the second team.

Dividing the Squad in two teams

a) Separate the players in two teams of 6 players each so that the condition above is satisfied.

Answer:

b) Find the number of ways to split the Azzurri into two teams as in part a, so that the players 10 and 11 end up on the same team.

Answer:
The Battistero has the shape of a regular octagon, so each of its 8 sides has the same length. Our friends Marco (M), Alessia (A) and Bruno (B) decide to help restore its floor. They share the job and each gets to work on various areas (see the figure on top).

**Side of the Battistero**

The sides of the right triangle shown in the picture are two meters long. Compute the length of the side of the octagon, by comparing the area of that triangle with the area of the square built on the side of the octagon.

**Answer:**

Who has more area to cover, Alessia or Bruno?

**Answer:**

Continues...
Bruno invites his friend Davide to join them in the restoration. Davide decides to help restoring the walls. That is not an easy task: the walls of the Battistero are 4 meters tall!

What fraction of the wall surface should Davide restore in order to cover exactly the same amount of area as Alessia?

**Answer:**
In Venice there are six islands labeled 1 to 6. Each pair of them is connected by a water canal flowing in only one direction (represented by an arrow).

**Counting Canals**

a. How many canals are in Venice?

**Answer:**

b. Alessia wants to start in island 1 and end up in island 6, by following the flow of the rivers and passing through each island exactly once. Can she do it?

**Answer:**

**What if we start and end where we want?**

Find and highlight a tour (if possible) passing through each island exactly once, following the flows. You may choose the starting and ending island.
We can change the “configuration” of Venice, by choosing the directions of the canals. That way we can design lots and lots of versions of Venice.

**Your version of Venice**

- Complete the picture on the left by deciding the river flow between every pair of islands. Can you find a tour passing through each island exactly once, by following the flows? **Highlight** a tour if you found one.

**How many versions of Venice?**

- How many different configurations of Venice can we create?

Answer:

![Different configurations of Venice](image)
Marco, Alessia and Bruno want to see the famous cities of Roma, Milano, Napoli and Firenze, a must-see for any tourist.

Our friends want to start their trip in Roma, but they can visit the remaining cities of Milano, Napoli and Firenze in any order. They are planning to stop by each city exactly once.

**Counting itineraries**

How many different itineraries are possible? (Roma – Firenze – Milano – Napoli is one such itinerary.)

**Answer:**

Continues...
To plan their itinerary, Alessia finds the following prices (in US dollars) for one-way flights. The length of each flight is also indicated. (The flight from Firenze to Napoli is long and expensive because the National Airline offers no direct flights between these two cities).

<table>
<thead>
<tr>
<th></th>
<th>ROM</th>
<th>FLO</th>
<th>MIL</th>
<th>NAP</th>
</tr>
</thead>
<tbody>
<tr>
<td>ROM</td>
<td></td>
<td>$85 1 hr</td>
<td>$130 1 hr</td>
<td>$90 1 hr</td>
</tr>
<tr>
<td>FLO</td>
<td>$85 1 hr</td>
<td></td>
<td>$130 1 hr</td>
<td>$230 3 hr</td>
</tr>
<tr>
<td>MIL</td>
<td>$130 1 hr</td>
<td>$130 1 hr</td>
<td></td>
<td>$85 1.5 hr</td>
</tr>
<tr>
<td>NAP</td>
<td>$90 1 hr</td>
<td>$230 3 hr</td>
<td>$85 1.5 hr</td>
<td></td>
</tr>
</tbody>
</table>

Recall: our friends want to visit each city once, starting in Rome.

**Cheapest itinerary**

- Find the **cheapest** possible itinerary.

**Fastest itinerary**

- Find the **fastest** possible itinerary.

Draw it on the map
Our friends continue their trip through Italy; Pisa is their next stop, where the Leaning Tower of Pisa is located, known worldwide for being unintentionally tilted.

Bruno sees an architect fixing the very top of the roof. Interested in estimating the inclination of the tower, Bruno wants to measure the difference between the length of the short side of the tower and its height.

The tag on the architect’s ladder reveals that the ladder is 60 meters long. This is not enough to solve Bruno’s inquiry. Luckily our friends manage to measure the shadows that the tower and the ladder leave on the ground (see the figure above).

**Height of the tower**

What is the height of the tower?

**Answer:**

Continues...
Short side of the tower

b. How long is the short side of the tower?

Answer:

Predicting the lean

c. In 1990 the Italian government performed restoration works on the tower of Pisa, because the leaning angle had reached 5.5 degrees and the tower was at serious risk of collapsing. Engineers measured that the top of the tower had been moving at a rate of \(\frac{1}{20}\) of an inch per year.

If in 1990 the top of the tower showed a 15 feet lean from where it used to be, in which year was the lean equal to 13.5 feet?

Answer:
Marco, Alessia and Bruno stop for breakfast at Caffe’ Aroma, a fancy coffee shop in Bologna famous for its croissants and for people watching. They decide to hang out there to see the habits of the local customers.

Every customer orders cappuccino and cornetto (croissant) for breakfast. Alessia notices that Italians like sugar in their cappuccino: it is either 1 packet of white sugar or 2 packets of brown sugar. Marco points out that 120 croissants and 155 packets of sugar have been consumed in the past hour.

**The Sugar Equations**

Let W and B represent the number of customers that choose white and brown sugar, respectively. Fill out the coefficients of the two equations below.

\[ _____ W + _____ B = 120 \]

\[ _____ W + _____ B = 155 \]
Brown sugar

b How many customers of Caffe’ Aroma used brown sugar in the past hour?

Answer:

Espresso

Before leaving the coffee shop, Bruno decides to try Espresso. This drink is served in 1 oz amounts. Bruno starts wondering whether there is more caffeine in a tiny 1 oz cup of espresso or in a large (8 oz) mug of American drip coffee. He finds out that the caffeine concentration is about 40 mg/oz (milligrams per ounce) in an espresso, and 12 mg/oz in driped coffee.

In her hometown in California, Alessia drinks three 8 oz. mugs of driped coffee every day.

How many espressos should she take in Italy to obtain the same amount of caffeine?

Recall: 120 croissants and 155 packets of sugar have been consumed in the past hour.

Did you know...

Espresso is by far the most popular coffee beverage in Italy. It is served in very small amounts (1 oz). Espresso is a very concentrated coffee.

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Espresso</td>
<td>40 mg/oz.</td>
</tr>
<tr>
<td>Driped</td>
<td>12 mg/oz.</td>
</tr>
</tbody>
</table>

Answer:
Music in The Duomo

A wild concert takes place in Milan

Eight professional musicians which are animals have made a visit to The Dome of Milan: an Anteater, a Beaver, a Coyote, a Duck, an Eagle, a Flamingo, a Gorilla and a Hare.

They will perform a musical concert, and each one will be located in one of the 8 places indicated in the picture (either a door or a window). There are 4 places on the left side: two windows on top, and two doors on the bottom. The right side follows the same pattern.

Marco, Bruno and Alessia are responsible for allocating the musicians. They receive the following requests from the animals:
Requests

1. Both the Anteater and the Beaver want to be on the left side.

2. The Coyote and the Hare want to be at the same height, but in opposite sides.

3. The Duck must lie directly below either the Anteater or the Flamingo.

4. The Beaver must be on the top.

5. The Flamingo wants to lie directly above the Hare or the Eagle.

6. The Eagle and the Beaver want to be placed at the same height.

7. The Anteater and the Hare would like to be as far as possible.

A second concert

b. The musicians decide to repeat the show. This time they only want the first five requests to be met. In how many different ways can you place the musicians?
Worried that Eric may not be familiar with grams and other units of measure, our friends use baker's math: they replace the amount of each ingredient by its percentage relative to the flour.

Example: for a recipe that requires 120oz of flour, 30oz of butter, 60oz of water and 60oz of sugar, one writes:

- Flour: 100%
- Butter: \((30/120) \times 100 = 25\%\)

Similarly, one has:
- Water: 50%.
- Sugar: 50%

Marco wants to send the following white bread recipe to Eric. Help Marco convert the recipe from standard measures into baker's math.

<table>
<thead>
<tr>
<th>White Bread</th>
<th>Weight (Kg)</th>
<th>Percentages</th>
</tr>
</thead>
<tbody>
<tr>
<td>Flour</td>
<td>50</td>
<td></td>
</tr>
<tr>
<td>Water</td>
<td>35</td>
<td></td>
</tr>
<tr>
<td>Salt</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>Yeast</td>
<td>0.5</td>
<td></td>
</tr>
</tbody>
</table>

Express your answers in the form “decimal %”, for example: 2.45%.
### Ciabatta

Alessia sends a message to Eric with the following recipe for **Ciabatta** (a short and long bread in the shape of a slipper). Help him get the recipe back to standard American measures, knowing that Eric wants to use 20 pounds of flour.

<table>
<thead>
<tr>
<th>Ciabatta</th>
<th>Weight (Pounds)</th>
<th>Percentages</th>
</tr>
</thead>
<tbody>
<tr>
<td>Flour</td>
<td>20</td>
<td>100%</td>
</tr>
<tr>
<td>Water</td>
<td></td>
<td>73%</td>
</tr>
<tr>
<td>Salt</td>
<td></td>
<td>1.1%</td>
</tr>
<tr>
<td>Yeast</td>
<td></td>
<td>1.8%</td>
</tr>
</tbody>
</table>

Would you be able to convert it back if the weight of the flour was not given?

### Slow Rise Baguette

Bruno sends Eric a recipe for slow rise baguette. He tries to convert the recipe from *baker’s math* to standard measures, but he is confused as no amount of flour is specified.

All he knows is that he needs to create 120 pounds of dough at the end. Help him convert the recipe in pounds.

<table>
<thead>
<tr>
<th>Slow rise baguette</th>
<th>Weight (pounds)</th>
<th>Percentages</th>
</tr>
</thead>
<tbody>
<tr>
<td>Flour</td>
<td></td>
<td>100%</td>
</tr>
<tr>
<td>Water</td>
<td></td>
<td>70%</td>
</tr>
<tr>
<td>Salt</td>
<td></td>
<td>2%</td>
</tr>
<tr>
<td>Yeast</td>
<td></td>
<td>0.75%</td>
</tr>
<tr>
<td><strong>Total dough</strong></td>
<td><strong>120</strong></td>
<td></td>
</tr>
</tbody>
</table>