Level 1 Math Circles
Beginning Probability

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Flipping Coins

Emma and Freddy are hanging out after school and flipping quarters. Freddy is about to flip another quarter.

1) How likely is it that the quarter lands heads up?

2) How likely is it that the quarter lands tails up?

3) What is the sum of these likelihoods?

Imagine that Emma flips a coin and it lands heads up. She then flips another coin and it also lands heads up. Emma then flips a third coin and it lands heads up. She is about to flip a fourth coin.

4) What is the likelihood that the fourth coin lands heads up?

5) What is the likelihood that the fourth coin lands heads up?

6) What is the sum of these likelihoods?

After Emma is finished flipping the fourth coin, Freddy collects the four coins and prepares to flip them.

7) How likely is it that all four quarters land heads up?

8) How likely is it that exactly three quarters land heads up?

9) How likely is it that exactly two quarters land heads up?

10) How likely is it that exactly one quarter lands heads up?

11) How likely is it that no quarters land heads up?

12) What is the sum of these likelihoods?

Emma and Freddy now have access to as many quarters as they like. Suppose Emma wants to flip n quarters.

13) What is the likelihood that all n quarters land heads up?

14) What is the likelihood that i of the n quarters land heads up?

15) What is the likelihood that the exact number of heads up quarters is less than or equal to i?

Flipping Unfair Coins

Emma and Freddy find a stash of n unfair coins. Assume that each coin has probability P of landing heads up.

1) If Freddy flips a coin, what is the chance that coin i lands tails up?

2) If Freddy flips all n coins, what is the chance that all coins land heads up?

3) If Freddy flips all n coins, what is the chance that exactly i of them land heads up?

4) If Emma flips a coin, what is the chance that coin i lands tails up?

5) If Emma flips all n coins, what is the chance that all coins land heads up?

6) If Emma flips all n coins, what is the likelihood that i consecutive coins land heads up?

Candy Time

Gwendolyn has a bowl of red, yellow, orange and green candies. There are 5 red candies, 3 yellow candies, 6 orange candies, and 2 green candies. Gwendolyn is about to randomly pick a candy out of the bowl.

1) How likely is it that she will pick a yellow candy?

2) What is the chance that the candy she will pick is red or green?

Suppose Gwendolyn puts the first candy back in the bowl and then decides she will randomly pick four candies out of the bowl.

3) What is the likelihood that all four candies will be orange?

4) What is the likelihood that all four candies will be green?

5) How likely is it that two of the four candies will be red?

6) What is the chances that the second and fourth candy chosen will be red?

7) How likely is it that none of the four candies will be neither yellow nor green?

Other Problems

1) Ingrid is showing Jennifer her three cards. One card is red on both sides, one card is green on both sides, and one card is red on one side and green on the other. Ingrid then puts the three cards behind her back. After shuffling the cards, Ingrid randomly picks a card and shows Jennifer one side of the card she picked. If the side shown to Jennifer is green, what is the likelihood that the other side of the card is green?

2) The USA is struck with a new deadly disease called Disease X. 1% of the population is infected with Disease X. However, the only way to know if someone has Disease X is to take a special medical test. However, this test is not perfect. If a sick person takes the test, there is a 98% chance it will return positive and and a 2% chance it will return negative. If a healthy person takes the test, there is a 97% chance it will return negative and a 3% chance that it will return positive. Kendrick lives in the USA and wants to know if he is sick. As a result, Kendrick takes the test and it comes back positive. What is the chance that Kendrick is sick?

3) Consider the game where you are given 50 white marbles and 50 black marbles. The player puts all 100 marbles in two separate bowls, but can split up the marbles however they choose. They are then blindfolded and the position of the bowls are randomized. While still blindfolded, the player picks a bowl and then picks a marble from the bowl. If the player picks a white marble, they win. If they do not pick a white marble, they lose. Lisa gets to play this game. How should Lisa distribute the marbles to maximize her chances of winning?