

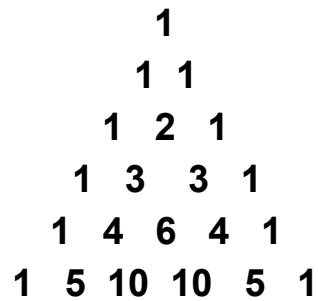
Logic Problem Solving

Logic problems tend to boil down to “Think logically and try everything until something sticks.” The trick to solving them is to break them down to their simplest parts. Don’t try to keep track of it all in your head, and have a plan.

The Plan

1. Read the problem/prompt carefully. Pay attention to the details
2. State what you’re trying to accomplish.
3. State what you need to know in order to reach that goal.
4. Break the problem down to its simplest parts.
 - a. Make a simpler/smaller scale equivalent if applicable.
 - b. Write down everything you know about every aspect of the problem
5. Draw diagrams and reframe the problem.
 - a. Say the problem aloud
 - b. Explain it to a friend
 - c. Look it over slowly while reading it aloud
 - d. Draw a picture
 - e. Make a model
6. Work towards solving the problem one step at a time while showing all your work and assumptions.
 - a. Look for patterns and trends
 - b. Look for obvious solutions
 - c. Try whatever comes to mind
7. Repeat steps 1-6 in varying orders until the problem is solved.

Example: Pascal's Triangle

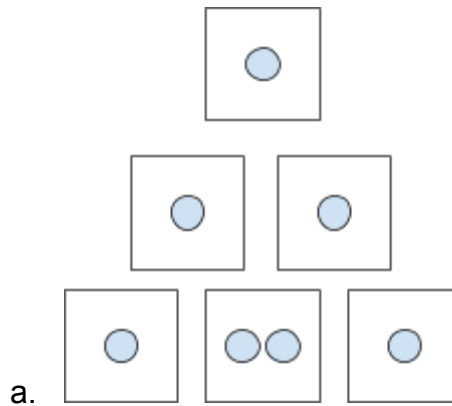


Find the next line in the sequence (let's solve this task together)

1. Read the problem/prompt carefully.
2. State what you're trying to accomplish.
 - a. I'm trying to find the next line in the sequence.
3. State what you need to know in order to reach that goal.
 - a. I need to know the pattern.
4. Break it down into its simplest parts/smallest scale

a. 1
1 1
1 2 1

5. Reframe the problem



6. Work towards solving the problem one step at a time while showing all your work and assumptions.
 - a. There's always a '1' on the edge
 - b. The inside has two dots; the sum of two ones.

c. What does the next line have?

i. 1
1 1
1 2 1
1 3 3 1

d. Two '3's. The sum of '1' and '2' which happen to be above each '3'.

e. Try adding '1' and '3' together, '3' and '3' together, and '3' and '1' together.

i. **"4 6 4"**

f. Compare to the next line in the sequence

i. 1 4 6 4 1

g. Hey, it fits! I just had to add '1' to the edges.

h. If I test the pattern for the rest of the sequence everything matches and I get '1

6 15 20 15 6 1' as the next line in the sequence

7. **'1 6 15 20 15 6 1'** is the next line in the sequence. Problem solved.

Additional questions for you:

- Label the rows of the Pascal triangle by indices $n=0, n=1, n=2, \dots$
- If **"1 X Y Y X 1"** is the row $n=20$,
 - Find x
 - Find y

Let's generalize this problem.

- If **"1 X Y Y X 1"** is the n^{th} row of the triangle, what are the values of x and y ?

- Now add the entries in each row. Do you notice any pattern?

- Using the above exercise; what is the sum of the elements on the 20th row of the triangle?

Practice: Triangle Two

2
2 2
2 4 2
2 8 8 2
2 16 64 16 2

What is the pattern?

Can you find the next row in this triangle?

Just like before, label the rows $n=0$, $n=1$, $n=2$ and so on. Compare the two Pascal triangles. How are the entries related?

More explicitly, if $1 \ x \ y \ z \ \dots \ z \ y \ x \ 1$ are the entries of the n^{th} row of the first Pascal triangle, what are the entries of the same row of the second triangle?

If $2 \ a \ b \ c \ \dots \ c \ b \ a \ 2$ is the 13^{th} row of the second triangle, what is the value of a ?

Practice: Morris' Triangle

1
1 1
2 1
1 2 1 1
1 1 1 2 2 1
3 1 2 2 1 1
1 3 1 1 2 2 2 1

What is the next line in the sequence? Use the space below to figure it out. Don't forget to work with those around you!

Practice: Yet Another Triangle

1
1 1
2 1
1 1 1 2
3 1 1 2
2 1 1 2 1 3

What is the next line in the sequence? Use the space below to figure it out. Don't forget to work with those around you!

Practice: The Monty Hall Problem

Suppose you're on a game show, and you're given the choice of three doors: Behind one door is a car; behind the others, goats. You pick a door, say No. 1, and the host, who knows what's behind the doors, opens another door, say No. 3, which has a goat. He then says to you, "Do you want to pick door No. 2?" Is it to your advantage to switch your choice?

Practice: Gnome Sequencing

10,000 gnomes are standing in an open space. They are all strictly identical : same hair, same clothes, same shoes... However, the colour of their hats is either red or blue. They don't know which color hat they're wearing, but they can see the color of the others' hats. They cannot communicate with each other in any form.

How can these gnomes separate each other in 2 groups of hat colours (blue & red)?

Practice: Flip Coins, Not Tables

You're in a dark room with 100 coins on a table, 12 are heads and the rest are tails, and the two faces are completely indistinguishable in the dark. How do you separate the coins into two piles so that the number of face up heads in each pile are equal?

Finale: Eye Color Problem

A group of people with assorted eye colors live on an island. They are all perfect logicians -- if a conclusion can be logically deduced, they will do it instantly. No one knows the color of their eyes. Every night at midnight, a ferry stops at the island. Any islanders who have figured out the color of their own eyes then leave the island, and the rest stay. Everyone can see everyone else at all times and keeps a count of the number of people they see with each eye color (excluding themselves), but they cannot otherwise communicate. Everyone on the island knows all the rules in this paragraph.

On this island there are 100 blue-eyed people, 100 brown-eyed people, and the Guru (she happens to have green eyes). So any given blue-eyed person can see 100 people with brown eyes and 99 people with blue eyes (and one with green), but that does not tell him his own eye color; as far as he knows the totals could be 101 brown and 99 blue. Or 100 brown, 99 blue, and he could have red eyes.

The Guru is allowed to speak once (let's say at noon), on one day in all their endless years on the island. Standing before the islanders, she says the following:

"I can see someone who has blue eyes."

Who leaves the island, and on what night?

There are no mirrors or reflecting surfaces, nothing dumb. It is not a trick question, and the answer is logical. It doesn't depend on tricky wording or anyone lying or guessing, and it doesn't involve people doing something silly like creating a sign language or doing genetics. The Guru is not making eye contact with anyone in particular; she's simply saying "I count at least one blue-eyed person on this island who isn't me."

And lastly, the answer is not "no one leaves."