WORKSHOP ON DYNAMICAL METHODS IN SPECTRAL THEORY OF QUASICRYSTALS

THE UNIVERSITY OF CALIFORNIA, IRVINE
DEPARTMENT OF MATHEMATICS
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Contents

Abstracts of Mini-Courses

Wannier Transform and Bloch Theory for Aperiodic Repetitive FLC Tilings

J. Bellissard
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1. Background: Delone sets, repetitivity, aperiodicity, finite local complexity, Hull, Transversal, the Lagarias group. Groupoids: the groupoid of the transversal, representation, covariance, covariant fields of Hilbert spaces, unitary equivalence.
2. The Anderson-Putnam complex, examples (Fibonnaci, Thue-Morse, Rudin-Shapiro, the octagonal lattice, the Penrose lattice), inflations (for the substitutive case or the general case), inverse limits and reconstruction of the Hull. Substitution tilings. Bratteli diagrams, horizontal edges, reconstruction of the groupoid of the transversal.
3. The Wannier transform for one AP-complex: quasi-momentum space, the periodic case (as a reminder), the case of FLC tilings, the Plancherel formula (the Annier transform as a unitary transformation between covariant fileds of Hilbert spaces).
4. The momentum space decomposition of the Schrödinger operator. Description of the boundary conditions, cohomological equation. Examples in one dimension, the higher dimensional case. The finite volume approximation: band spectrum, absolute continuity. Discussion and conjectures: the infinite volume limit. Renormalization and inverse limit, the substitution case, expected estimates. A conjecture: absolutely continuous spectrum in $D > 2$ in the perturbative regime, a route towards a proof.

Trace Map Dynamics and spectral properties of the Fibonacci Hamiltonian.

D. Damanik & A. Gorodetski
Rice University & UC Irvine
Department of Mathematics

1. Discrete one dimensional Schrödinger operators. Quantum dynamics, spectral measures, solutions and transfer matrices. Fibonacci Hamiltonian, recursions, Fibonacci Trace Map. Suto's Theorem.
2. Hyperbolic Dynamical Systems. Hyperbolicity of the Trace Map. Dynamically defined Cantor sets, their properties. Spectrum of the Fibonacci Hamiltonian as a dynamically defined Cantor set.
3. Fractal dimension of the spectrum. Gap labeling and gap opening. Transport exponents. Density of states measure. Square and cubic Fibanacci Hamitonian. Structure of spectrum and density of states measure.
4. Sums of Cantor sets and convolutions of singular measures. Absolute continuity of the density of states measure of square Fibonacci Hamiltonian in small coupling regime. Known results and open questions. Spectral Theory of self-similar lattices and dynamics of rational maps.

Spectral theory and dynamics of quasi-periodic Jacobi cocycles

S. Jitomirskaya & C. Marx
UC Irvine & California Institute of Technology
Department of Mathematics

Recently, several problems from the spectral theory of 1 dimensional quasi-periodic Jacobi operators could be successfully solved studying the dynamics of associated cocycles. Even though some of these ideas are well known in dynamical systems, they are still less common in the spectral theory community. In our lectures, we aim to bridge the gap, presenting a survey of the dynamics of quasi-periodic cocycles from a spectral theorists point of view.

Topics that will be discussed include: quasi-periodic Jacobi and higher-dimensional cocycles, Lyapunov exponent (LE) and Oseledets' theorem, Conjugacies - Aubry duality revisited, dominated splittings, (almost) reducibility and the absolutely continuous spectrum, complexified cocycles, continuity of the LE, Avila's global theory and spectral consequences. Our recent work on Extended Harper's Model, a generalization of almost Mathieu, will serve as an illustration for some of the presented methods.

Spectral Theory of Self-Similar Lattices and Dynamics of Rational Maps

C. Sabot
Université Lyon 1

These lectures will focus on the spectral theory of "finitely ramified" self-similar lattices and their continuous counterparts, self-similar fractals (e.g. Sierpinski gasket, Snowflake, etc). These lattices, which can be viewed as toy-models for quasi-crystals, exhibit interesting relations with the dynamics of rational maps with several variables. The first lectures will be devoted to elementary properties of these lattices and to a short overview of multidimensional rational dynamics and pluripotential theory. Then, we will describe the renormalization map associated with a self-similar lattice and the crucial point of its appropriate compactification. We relate some characteristics of the dynamics of its iterates with some characteristics of the spectrum of our operator. More specifically, we give an explicit formula for the density of states in terms of the Green current of the map, and we relate the indeterminacy points of the map with the so-called Neuman-Dirichlet eigenvalues which lead to eigenfunctions with compact support on the unbounded lattice. Depending on the asymptotic degree of the map the spectral properties of the operators are drastically different.

Abstracts of Talks

Dynamics of Unitary Operators

J. Fillman
Rice University
Department of Mathematics

We discuss relationships between spectral characteristics of unitary operators and the dynamical description of spreading of a corresponding quantum walk over an orthonormal basis. The primary result is a natural extension of the Guarneri-Combes-Last bound: uniform Hölder continuity of a spectral measure implies quantitative lower bounds on the spreading of the corresponding wave packet. Together with results of Damanik, Munger, Ong, and Yessen, this allows us to prove such lower bounds for the spreading of the quantum walk corresponding to a CMV matrix which has Fibonacci Verblunsky coefficients.

Inverse Resonance Problem for Perturbations of Periodic Jacobi Matrices: Existence, Uniqueness, and Stability

R. Kozhan
UC Los Angeles
Department of Mathematics

By establishing the explicit form of the spectral measure, we are able to solve the inverse resonance problem for super-exponentially decaying perturbations of periodic Jacobi matrices.

Continuity of the Lyapunov Exponents for Quasi-Periodic Co-cycles

S. Klein
CMAF, Universidade de Lisboa, Portugal

Consider the Banach space of real analytic linear co-cycles with values in the general linear group of any dimension and base dynamics given by a Diophantine translation on the circle.

We prove a precise higher dimensional Avalanche Principle and use it in an inductive scheme to show that the Lyapunov spectrum blocks associated to a gap pattern in the Lyapunov spectrum of such a co-cycle are locally Holder continuous. We also show that all Lyapunov exponents are continuous everywhere in this Banach space, irrespective of any gap pattern in their spectrum. Moreover, our results apply to other quasi-periodic base dynamics, albeit with a loss in the modulus of continuity.

[This is joint work with Pedro Duarte.]

Spectral Decimation and Complex Dynamics: Laplacians on Self-Similar Fractals and their Spectral Zeta Functions

M. L. Lapidus
UC Riverside
Department of Mathematics

We investigate the spectral zeta functions of certain fractal differential operators, such as fractal Sturm-Liouville operators acting on a self-similar interval (equipped with a given self-similar measure and Dirichlet form) and the fractal Laplacian on the unbounded (or infinite) gasket. In each case, using the so-called spectral decimation method and its extension to several complex variables by C. Sabot, we obtain a factorization of the corresponding spectral zeta function expressed (in general) in terms of a suitable hyperfunction, a geometric zeta function, and the iteration of a multi-variable rational function acting in complex projective space. The resulting factorization formula extends earlier work of the authors for fractal strings, later itself extended by A. Teplyaev to the Laplacian on the finite (or bounded) Sierpinski gasket. This talk is based on joint work with Nishu Lal (in a research article in J. Phys. A: Math. and Theor., 2012, and a survey article in Contemp. Math., vol. 601, in press, 2013).

Bounded Variation Conditions and Absolutely Continuous Spectrum

M. Lukic
Rice University
Department of Mathematics

For one-dimensional Schrödinger operators it is known that, on $(0, \infty)$, L^1 potentials have purely a.c. (absolutely continuous) spectrum and L^2 potentials have a.c. spectrum. Analogous results hold for Jacobi and CMV matrices. In this talk, we will discuss a growing body of results which show that, similarly, various L^1 or L^2 variation conditions on the potential imply purely a.c. spectrum or a.c. spectrum on intervals away from certain critical points. We will also present some new results on the behavior of the spectral density near the critical points.

Upper bounds on quantum dynamics for quasiperiodic Schrödinger operators with rough potentials

R. Mavi
University of Virginia
Department of Mathematics

While delocalization for Hamiltonians is forced by continuity of the spectral measure under general conditions, this is only a part of the picture as singular spectrum cannot enforce dynamical localization. General conditions for upper bounds on quantum dynamics have been developed by various authors, notably by Killip, Kiselev and Last and Damanik and Tcheremchantsev. As with most theories for quasiperiodic Schrödinger operators, complete results have been restricted to the cases where the potential is analytic or finite valued. Our focus in this talk is to extend some of these results to Schrödinger operators on $\ell^2(\mathbb{Z})$ with rough (and even discontinuous) quasiperiodic potentials. This talk covers joint work with Svetlana Jitomirskaya.

Spectra of Primitive Invertible Substitution Schrödinger and Jacobi Operators

M. Mei
UC Irvine
Department of Mathematics

In a series of papers, Damanik and Gorodetski have investigated spectral properties of the Fibonacci Hamiltonian: fractal dimension, gap structure, and exact dimensionality of the integrated density of states. In this talk, we generalize these results to potentials given by primitive invertible substitutions on two letters. We also announce results (joint with W. Yessen) generalizing similar spectral results by Yessen for the tridiagonal Fibonacci Hamiltonian to Jacobi operators whose potential and Laplacian terms are moderated by a primitive invertible substitutions on two letters.

Spreading Estimate for a Quantum Walk on $\mathbb{Z}$ with Fibonacci Coins

P. Munger
Rice University
Department of Mathematics

We give estimates for the spreading rates of quantum walks on the line with time-independent coins following a Fibonacci sequence. The estimates are explicit in terms of the parameters of the system.

Limit stochastical differential equations (SDEs) for products of random matrices in a critical scaling

C. Sadel
University of British Columbia, Canada
Department of Mathematics

We consider the Markov process given by products of i.i.d. random matrices that are perturbations of a fixed non-random matrix and the randomness is coupled with some small coupling constant. Such random products occur in terms of transfer matrices for random quasi-one dimensional Schroedinger operators with i.i.d. matrix potential. Letting the number of factors going to infinity and the random disorder going to zero in a critical scaling we obtain a a limit process for a certain Schur complement of the random products. This limit is described by an SDE (stochastical differential equation). This allows us to obtain a limit SDE for the Markov processes given by the action of the random products on Grassmann manifolds. This is a joint work with Balint Virag.

Mixed Behavior in the Fibonacci Trace Map

W. Yessen
UC Irvine
Department of Mathematics

The Fibonacci trace map – a polynomial map on the three-dimensional Euclidean space – is known to exhibit rich dynamical behavior. Some of its dynamical properties (namely, hyperbolicity) have been exploited to a great extend in investigation of the spectrum of the Fibonacci Hamiltonian (the Schrödinger operator with potential generated by the Fibonacci substitution). We shall describe another phenomenon, which also shows up in some physical models, but has not been rigorously investigated hitherto – namely, conservative homoclinic phenomena producing mixed behavior similar to that observed in the well known Taylor-Chirikov standard map. This type of behavior (and some very famous questions that are still open about such systems) is of interest in mathematical physics as well as in dynamical systems. For example, this behavior has been recently observed in the Fibonacci quantum walks.

Singular IDS for One-Frequency Quasi-Periodic Schrödinger Operators

Z. Zhang
Northwestern University
Department of Mathematics

In this talk, I will show that the integral density of states (IDS) of one-frequency quasi-periodic Schrödinger operators, for any fixed irrational frequency, is singular continuous for a generic subset of $C^0(\mathbb{R}/\mathbb{Z}, \mathbb{R})$. I will also show that in the space, $(\mathbb{R}/\mathbb{Z}) \times C^0(\mathbb{R}/\mathbb{Z}, \mathbb{R})$, of frequencies and potentials, there is a dense subset such that the corresponding IDS is singular continuous while the spectrum contains non-degenerate intervals. This is joint work with Artur Avila and David Damanik.