6) \( x_0 \in (a, b) \). Then every ball \( B(x_0, \varepsilon) \) contains points of \((a, b)\), for example, let \( \delta = \min \{ x_0 - a, b - x_0 \} \)

\[ a \xrightarrow{\delta} x_0 \xrightarrow{\varepsilon} b \]

and let \( \delta' = \min \{ \delta, \varepsilon \} \).

Then \( B(x_0, \delta') \subseteq B(x_0, \varepsilon) \), and

\[ x_0 + \frac{\delta'}{2} \in B(x_0, \delta'). \]

Therefore \( x_0 \in \) derived set of \((a, b)\).

To see that \( a \in \) derived set of \((a, b)\),

for any \( \varepsilon > 0 \), \( B(a, \varepsilon) \) contains \( a + \frac{1}{n} \) for \( n \) large enough, and for \( \frac{1}{n} < b - a \),

\[ a < a + \frac{1}{n} < b \Rightarrow a + \frac{1}{n} \in B(a, \varepsilon). \]

7, 8, 9 | Already posted

10 | Properties 1) 2) 3) are trivial. For 4), observe that the inequality holds if \( x = z \) (since then, \( d(x, z) = 0 \)), and if \( x \neq z \), \( d(x, z) = 1 \), and for any \( y \in \mathbb{X} \),

either \( y \neq x \) or \( y \neq z \) (by transitivity of "\( = \)"),

in which \( d(x, y) + d(y, z) \geq 1 \).

12 | Since \( d(x, y) = 1 \) for \( x \neq y \), \( \mathbb{B}(x, \frac{1}{2}) = \{ x \} \), for every \( x \).

Therefore \( \mathcal{C} \) contains all singletons \( \{ x \} \), and

hence \( \mathcal{U} \) is the discrete topology on \( \mathbb{X} \).