Stability Conjectures in K-Theory and Modules over Semisimple Skew Group Rings

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This talk will focus on a problem from algebraic K-theory that finds part of its solution in group cohomology. In 2003 Carlsson conjectured that the completed algebraic K-theory spectrum of a non-algebraically closed field can be obtained from the ring of semilinear representations of its absolute galois group [1]. Verification of this conjecture requires an understanding of the module category of a skew group ring.

Semisimple skew group rings are products of central simple algebras over fields and thus correspond to elements of the Brauer group of a product of fields. We will show that the element of the Brauer group corresponding to a particular skew group ring $E\langle G\rangle$, where E is a field and G is a group acting on E with kernel N, can be computed using a cocycle in the cohomology group $H^2(Gal(E/E^G), N)$. This group also arises in Galois embedding problems.

In this talk I'll start by developing the K-theoretic motivation for this problem without assuming any prior knowledge of the subject. Then I'll explain how to find the irreducible $E\langle G\rangle$-modules and their tensor-product structure (this is joint work with Ken Ribet). Finally, I'll state a few corollaries and work some examples.

References

[1] G. Carlsson, *Structured stable homotopy theory and the descent problem for the algebraic k-theory of fields*, preprint, 2003.

[2] I. Reiner, *Maximal orders*, Academic Press [A subsidiary of Harcourt Brace Jovanovich, Publishers], London-New York, 1975, London Mathematical Society Monographs, No. 5.