Red Lodge Conference

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Galois Representations with Prescribed Ramification

Deligne proved that a cuspidal modular form f which is an eigenform for the Hecke algebra gives rise to a system of Galois representations $\rho_{f,l}\colon \mathrm{Gal}(\mathbf{Q}/\mathbf{Q}) \to \mathrm{GL}_2(\overline{\mathbf{Q}}_l)$. For each prime p, one can study the associated Weil-Deligne representation of the inertia group

$$\rho_p\colon I_p \to \mathrm{GL}_2(\mathbf{C}).$$

We ask the following inverse problem: Given an integer $k \geq 2$ and a collection of representations ρ_p as above, all but finitely many of which are trivial, can one always find a cusp form f of weight k whose associated Galois representation matches ρ_p when restricted to the inertia groups for each p? To attack this question, we will first observe that the possible families $\{\rho_p\}$ coming from cusp forms of a given level has everything to do with the irreducible representations of $\mathrm{SL}_2(\mathbf{Z}/N\mathbf{Z})$ appearing in $S_k(\Gamma(N))$. We will then apply an equivariant version of the Riemann-Roch formula to the modular curve $X(N)$ to count the number of desired cusp forms.

References

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