TOWER OF COARSE MODULI

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Modular Towers is introduced by M. Fried for analogue of the towers of modular curves in the framework of Hurwitz space theory. Let p be a prime, G a p-perfect (admits no $\mathbb{Z}/p$ quotient) finite group such that $|G|$ is divisible by p, $\mathbf{C}$ be r-tuple of conjugacy classes of which order of the elements is prime to p. Then we can consider the tower of Hurwitz space associated to the data $(p, G, \mathbf{C})$, $(\mathcal{H}_{n+1} \to \mathcal{H}_n)_{n\geq 0}$. Here $\mathcal{H}_n$ is moduli stack (algebraic stack) and not always have fine moduli but coarse moduli H_n. PGL_2 acts on H_n canonically and we get reduced Hurwitz space H_n^{rd} and reduced modular tower $(H_{n+1}^{rd} \to H_n^{rd})_{n\geq 0}$ as the quotient. There are diophantine problem about modular towers and some is solved in $\mathcal{H}$ case in [1]. I considerd the difference between $\mathcal{H}$ and H^{rd} using some group theory in [3].

The theorem proved in [1] is the following.

Theorem 0.1. *Let K be a number field. Then,*

$$\varprojlim \mathcal{H}_n(K) = \emptyset$$

In other words of RIGP, it is equivalent to that some profinite group called the universal p-Frattini cover of G can not be regularly realized in finite number of branch points. More general result is obtained in [2] using abelialization.

The goal of this talk is the following theorem.

Theorem 0.2. *Let K be a number field. Then,*

$$\varprojlim H_n^{rd}(K) = \emptyset$$

References

[1] P. Bailey and M. D. Fried Hurwitz monodromy, spin separation and higher levels of a modular tower. Arithmetic fundamental groups and noncommutative algebra (Berkeley, CA, 1999), 79–220, Proc. Sympos. Pure Math., 70, Amer. Math. Soc., Providence, RI, 2002.

[2] A. Cadoret, Rational points on Hurwitz Towers, preprint as of Jan. 2005.

[3] K. Kimura Modular Towers for finite groups that may not be centerfree, preprint as of Apr. 2005. http://mathmaticfreestyle.up.seesaa.net/image/modular.dvi