GENERALIZED FUCHSIAN GROUPS AND THE p-REDUCTION THEORY OF ELEMENTS IN HURWITZ SPACES

THOMAS WEIGEL

Abstract

Fuchsian groups of the first kind are precisely the subgroups Γ of $PGl_2(\mathbb{R})$ which act discontinuously and co-compactly on the hyperbolic plane $\mathbb{H}^2$. The fundamental domain constructed by R.Fricke can be used to construct a combinatorical 2-dimensional cell complex $C(\Gamma)$ on which Γ acts cellularly, faithfully, co-compactly, and with finite stabilizers. Furthermore, its geometric realization $|C(\Gamma)|$ is connected and contractable (cf. [We06]). Roughly speaking - a *generalized Fuchsian group* Γ is a group for which there exists such a cell complex.

On one hand one can think of generalized Fuchsian groups as a 2-dimensional analogue of finitely generated virtually free groups, which act co-compactly and with finite stabilizers on a tree (cf. [Se80]). On the other hand the two kinds of groups which underlay behind the "2 Frattini principles" of M.D.Fried (cf. [Fr06, §3.2]) are - although very different - 2 standard types of generalized Fuchsian groups. But, they are not Fuchsian.

So apart from the group theoretic and topological properties of generalized Fuchsian groups, we will also discuss the role they play in the p-reduction theory of Hurwitz spaces of a finite p-perfect group G. In particular, the distinction between p-cusps, g-p'- and o-p'-cusps in [Fr06, §3.2] is related to the two different p-reduction principles one can put in place.

References

[Fr06] M.D. Fried, *The Main Conjecture of Modular Towers and its higher rank generalization*, in Groupes de Galois arithmetiques et differentiels (Luminy 2004; eds. D. Bertrand and P. Dèbes), Sem. et Congres, Vol. 13, 2006. At www.math.uci.edu/ mfried/talkfiles/lum-fried12-12-05.html

[Se80] J-P. Serre, *Trees*, Springer-Verlag, Berlin 1980.

[We06] T. Weigel, *Fricke domains and explicit Poincaré duality*, in preparation.

Th. Weigel, Università di Milano-Bicocca, U5-3067, Via R.Cozzi, 53, 20125 Milano, Italy

E-mail address: thomas.weigel@unimib.it

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