

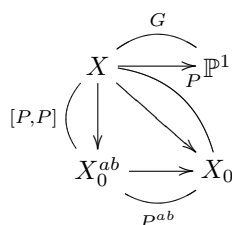
# Arithmetic properties of Moduli spaces for $p$ -etale $G$ -covers and torsion on abelian varieties

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## Abstract

Fix a finite group  $G$  and a prime  $p$  dividing  $|G|$ . A  $G$ -cover  $X \rightarrow \mathbb{P}^1$  with group  $G$  is said to be  $p$ -etale if it factors through an etale  $G$ -cover  $X \rightarrow X_0$  with group a  $p$ -subgroup  $P$  of  $G$ .



Let  $\mathbf{C}$  be the inertia canonical invariant of  $X \rightarrow \mathbb{P}^1$ ,  $g = g(G, \mathbf{C})$  be the genus of  $X$  and  $g_0^{ab} = g(P, \mathbf{C})$  be the genus of  $X_0^{ab}$ . Write  $H_{g, \mathbf{C}}$  (resp.  $H_{g_0^{ab}}$ ) for the coarse moduli space of  $G$ -curves with genus  $g$  (resp.  $g_0$ ) whose resulting  $G$ -cover has group  $G$  and inertia canonical invariant  $\mathbf{C}$  (resp. is etale with group  $P^{ab}$ ). There is a natural morphism  $H_{g, \mathbf{C}} \rightarrow H_{g_0^{ab}}$  corresponding to the functor sending  $X \rightarrow \mathbb{P}^1$  to  $X \rightarrow X_0^{ab}$  and which, composed with the Torelli morphism, yields a morphism  $H_{g, \mathbf{C}} \rightarrow A_{g_0^{ab}}$ .

Rational points on  $H_{g, \mathbf{C}}$  are connected to torsion on abelian varieties in isogeny classes of rational points in the image of  $H_{g, \mathbf{C}} \rightarrow A_{g_0^{ab}}$ . This observation yields new insights in the theory of modular towers. For instance, it shows that Fried's conjecture for modular towers is a special case of the strong torsion conjecture for abelian varieties or that there is no projective system of  $\mathbb{Q}^{ab}$  rational points along modular towers.

After exposing these results, I will focus on dihedral towers and related conjectures. I will prove the dihedral conjecture over  $\mathbb{Q}^{ab}$  and I will give a transcendental uniformization of the 3-dimensional dihedral tower, obtaining in particular that the  $n$ th level of this tower is of general type for  $n$  large enough.