Arithmetic properties of Moduli spaces for \( p \)-etale G-covers and torsion on abelian varieties

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Arithmetic Galois Theory and Moduli Spaces

Abstract

Fix a finite group \( G \) and a prime \( p \) dividing \( |G| \). A G-cover \( X \to \mathbb{P}^1 \) with group \( G \) is said to be \( p \)-etale if it factors through an etale G-cover \( X \to X_0 \) with group a \( p \)-subgroup \( P \) of \( G \).

Let \( C \) be the inertia canonical invariant of \( X \to \mathbb{P}^1 \), \( g = g(G, C) \) be the genus of \( X \) and \( g_0 = g(P, C) \) be the genus of \( X_0^{ab} \). Write \( H_{g, C} \) (resp. \( H_{g_0}^{ab} \)) for the coarse moduli space of \( G \)-curves with genus \( g \) (resp. \( g_0 \)) whose resulting G-cover has group \( G \) and inertia canonical invariant \( C \) (resp. is etale with group \( P^{ab} \)). There is a natural morphism \( H_{g, C} \to H_{g_0}^{ab} \) corresponding to the functor sending \( X \to \mathbb{P}^1 \) to \( X \to X_0^{ab} \) and which, composed with the Torelli morphism, yields a morphism \( H_{g, C} \to A_{g_0}^{ab} \).

Rational points on \( H_{g, C} \) are connected to torsion on abelian varieties in isogeny classes of rational points in the image of \( H_{g, C} \to A_{g_0}^{ab} \). This observation yields new insights in the theory of modular towers. For instance, it shows that Fried’s conjecture for modular towers is a special case of the strong torsion conjecture for abelian varieties or that there is no projective system of \( \mathbb{Q}^{ab} \) rational points along modular towers.

After exposing these results, I will focus on dihedral towers and related conjectures. I will prove the dihedral conjecture over \( \mathbb{Q}^{ab} \) and I will give a transcendental uniformization of the 3-dimensional dihedral tower, obtaining in particular that the \( n \)th level of this tower is of general type for \( n \) large enough.