Arithmetic properties of Moduli spaces for p-etale G-covers and torsion on abelian varieties

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Abstract

Fix a finite group G and a prime p dividing $|G|$. A G-cover $X \to \mathbb{P}^1$ with group G is said to be *p-etale* if it factors through an etale G-cover $X \to X_0$ with group a p-subgroup P of G.

$$\begin{tikzcd}
X \arrow[r] \arrow[r, bend left, no head, "G"] \arrow[d] \arrow[d, bend right, no head, "{[P,P]}"'] \arrow[dr] \arrow[dr, bend left, no head, "P"] & \mathbb{P}^1 \\
X_0^{ab} \arrow[r] \arrow[r, bend right, no head, "P^{ab}"'] & X_0
\end{tikzcd}$$

Let $\mathbf{C}$ be the inertia canonical invariant of $X \to \mathbb{P}^1$, $g = g(G, \mathbf{C})$ be the genus of X and $g_0^{ab} = g(P, \mathbf{C})$ be the genus of X_0^{ab}. Write $\mathrm{H}_{g,\mathbf{C}}$ (resp. $\mathrm{H}_{g_0^{ab}}$) for the coarse moduli space of G-curves with genus g (resp. g_0) whose resulting G-cover has group G and inertia canonical invariant $\mathbf{C}$ (resp. is etale with group P^{ab}). There is a natural morphism $\mathrm{H}_{g,\mathbf{C}} \to \mathrm{H}_{g_0^{ab}}$ corresponding to the functor sending $X \to \mathbb{P}^1$ to $X \to X_0^{ab}$ and which, composed with the Torelli morphism, yields a morphism $\mathrm{H}_{g,\mathbf{C}} \to \mathrm{A}_{g_0^{ab}}$.
Rational points on $\mathrm{H}_{g,\mathbf{C}}$ are connected to torsion on abelian varieties in isogeny classes of rational points in the image of $\mathrm{H}_{g,\mathbf{C}} \to \mathrm{A}_{g_0^{ab}}$. This observation yields new insights in the theory of modular towers. For instance, it shows that Fried's conjecture for modular towers is a special case of the strong torsion conjecture for abelian varieties or that there is no projective system of $\mathbb{Q}^{ab}$ rational points along modular towers.
After exposing these results, I will focus on dihedral towers and related conjectures. I will prove the dihedral conjecture over $\mathbb{Q}^{ab}$ and I will give a transcendental uniformization of the 3-dimensional dihedral tower, obtaining in particular that the nth level of this tower is of general type for n large enough.