Curves of genus 2 with elliptic differentials and associated Hurwitz spaces

In the lecture we discuss the Hurwitz space related to covers

$$\varphi : \mathbb{P}^1_K \to \mathbb{P}^1_K$$

of degree n prime to $\mathrm{char}(K)$ ramified in 5 points $P_1, \ldots, P_5$ with ramification order at most 2 such that the ramification cycle corresponding to P_5 in the Galois closure of the cover is a transposition and explain that it is isomorphic to the moduli space related to covers

$$f : C \to E$$

where C is a curve of genus 2, E is an elliptic curve and f is a normalized morphism of degree n.
This result enables us to combine methods from group theory and from algebraic and arithmetic geometry to get results about diophantine properties of the moduli spaces like rigidity numbers, special curves and rational points closely related to conjectures (and results) about elliptic curves with Galois-isomorphic torsion structures.