ARITHMETIC GALOIS THEORY AND RELATED MODULI SPACES

“Galois Cohomology, Quotients of Absolute Galois Groups, and A Little Modular Representation Theory”

ABSTRACT. Absolute Galois groups of fields are mysterious. One can try to find manageable but still non-trivial quotients of absolute Galois groups. Let \( p \) be a prime number. In joint work with D. Benson, N. Lemire, and J. Swallow we consider groups \( T(E/F) = G_F/\Phi(G_E) \), where \( F \) is a field containing a primitive \( p \)th root of unity such that its absolute Galois group \( G_F \) is a pro-\( p \) group, \( E/F \) is a cyclic extension of degree \( p \) and \( \Phi(G_E) \) is the Frattini subgroup of \( G_E \). We determine all possible groups \( T(E/F) \). Further assuming the Bloch-Kato conjecture we determine the \( \mathbb{F}_p[G_F/G_E] \)-module \( H^i(G_E, \mathbb{F}_p) \) for all \( i = 1, 2, \ldots \) which extends the previous work of Borevič and Faddeev on the \( \mathbb{F}_p[G_F/G_E] \) structure \( H^1(G_E, \mathbb{F}_p) \) in the case of local fields.