Lines in the Curriculum: A persistent obstruction to Achievement connecting Algebra and Geometry
11 AM, March 29, 2010: Conference Room at MIND Institute

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March 29, 2010
Summary

Today’s Problem: Use 9th grade algebra and 10th grade geometry to describe lines in 3-space. A student who understands lines, understands a lot.

Part I: Description of Lines in the K-14 Curriculum
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1 Part I: Description of Lines in the K-14 Curriculum
   I.A: There are many lines in the curriculum
   I.B: The Point of Lines: Direction and Orientation
   I.C: What is a line?: A Persistent Curricular Problem
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2. **Part II:** California Framework Awareness?
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2. **Part II:** California Framework Awareness?
   - II.A. Where the Framework stood at the end of the '90s
   - II.B. What are lines worth?
   - II.C. The Howard Thompson Story
   - II.D. Standards and Curricular Goals
Rubric: What is a line in —?

[In practice the material is usually taught earlier than given here.]

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3. In 8th-9th grade? Answer: An expression like $y = 2x + 3$. 
4. In 10th grade? Answer: It is undefined, except by properties: Two points determine a unique line.
5. In 11th-12th grade? Answer: An expression like $y = mx + b$. 

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2. In 1st year Calculus/physics? Answer: $y = mx + b$: $m = \frac{d}{dx}(f(x))|_{x=x_0}$ for $f(x)$ at a point $x_0$. Approximating 4th grade line to the graph of $f$ near $x_0$. 
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   Approximating 4th grade line to the graph of $f$ near $x_0$.

3. In 2nd year Calculus or in physics? Answer:
   \[ \{(x_0, y_0, z_0) + t(u, v, w) \mid t \in \mathbb{R}\}. \]
   A *parametric line*: $t$ is a *parameter*. 

Mike Fried | Lines in the Curriculum
Suppose I want to guide you to somewhere where you are walking across a field. I might use time and direction.

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I.B: The Point of Lines: Direction and Orientation

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- How would I explain this to you? Answer: I might use my left arm to point. Maybe you are on an airplane (or a proton) and I’m directing you to a target up in the air.
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- What picture might I have?: Answer: At time \(t = 0\) you are at \((x_0, y_0, z_0)\). You end up in one minute at \((x_0 + u, y_0 + v, z_0 + w)\), in 2 minutes at \((x_0 + 2u, y_0 + 2v, z_0 + 2w)\), 3 minutes, etc.
If \( y = 2x + 3 \) is a line in \((x, y)\)-space, then is \( z = 2x + 3y + 4 \) a line in \((x, y, z)\)-space?

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4. From Euclid: If two points are in a plane, the line determined by them has all its points (lies) on the plane.
   - 2nd tack: Find points on $S = \{ (x, y, z) \mid z = 2x + 3y + 4 \}$. What is the meaning of taking $x = 0$, then taking $y = 0$?
Using the principle: Two points determine a line.

1. Setting \( x = 0 \) confines points on \( S \) to a coordinate plane.

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S_{x=0} = \{(0, y, z) \mid z = 3y + 4\}.
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Same with \( y = 0 \). Do the points so confined look like lines?
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2. In each coordinate plane there is a traditional 9th grade line. Two distinct (not parallel) lines on $S$ meet at a point on $S$.

3. Does $x$, $y$ and $z$ appearing to first power give $S$ one property of a plane? Example: The points $(1, 1, 9)$ and $(-1, -1, -1)$ are on $S$. Then, all points on the line
   \[ L((1, 1, 9) : (-1, -1, -1)) \]

   they determine are also on $S$. How would you check? Hint: How would you write expressions for those points?
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5. How students perform: With similar questions, students bring out slopes. Such training won’t support 3-D thinking.
Mathematica produces $S$

Two lines on a Plane

Line $Sx = 0$

$\downarrow$

Line $Sy = 0$
Mathematica produces $S$
Two lines on a Plane

Line $S_{x=0}$ →
Mathematica produces $S$

Two lines on a Plane

Line $S_{x=0}$

$\downarrow$ Line $S_{y=0}$

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Lines in the Curriculum
In 218 page Framework, here are all references to lines or planes:

- p. 83: "In this strand [Geometry] students . . . investigate 2-dimensional and 3-dimensional space by exploring shape, area and volume; lines, angles, points and surfaces."
II. A. Where the Framework stood at the end of the ’90s

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- **p. 126**: "maps in middle school involved plane geometry. "in their work with maps, students explore . . . paths and how to specify straight paths by . . . [using] . . . distance."
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- p. 143: "Students need to see the analogy between familiar frames of reference used to locate places (street patterns, building interiors . . .) and . . . coordinate geometry."
- Yet, it was tessellations, packing problems and fractals that were brought up as relevant topics. The text didn’t mention lines, and the difficulties in representing them in 3-space.
II.B. What are lines worth?

Strongest statement relating HS courses, pps 155–157: 3 headings

Connections between functions and algebra
Connections between functions and geometry
Connections between algebra and geometry

Lines were never mentioned as a persistent difficulty.

- 2nd Year Calculus [seems to stand] all alone.
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- Result: Almost all students fail: Fail to see how mathematics and engineering, physics, chemistry, economics, social sciences – that use line as a tool – relate.
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- Another day’s Lesson: Parametric lines give meaning to solving equations. Even if you don’t have a method to solve particular equations, there still is a well-defined meaning to solving them.
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II.C. The Howard Thompson Story

http://www.math.uci.edu/ mfried/edlist-tech/gold02-08-98.html

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- Sloan Foundation Grant toward an acknowledged problem: *Nearly* 100% minority student wipeout from 1st quarter sophomore Calculus.
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- My statistic: 51 "black" [our word then] students took the 1st quarter vector calc. in a 10 year period; one got through.
An enlightened administrator

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- My answer, "Howard Thompson." Dennis repeated that, "Howard Thompson? He was the only black student who got through the first quarter of vector calculus?"
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II.D. Standards and Curricular Goals

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- Result: States set their standards at widely varied levels. Obama would measure each student’s academic growth, regardless of their starting performance level.
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- Analogy: The Greeks were brilliant and so everything marched forward from that time.
- Action: Common-standards effort has produced a draft.
- Assessment is everything: That requires replacing NCLB’s much-criticized school rating system, known as *adequate yearly progress*, with a new accountability system.