1. Background and Purpose

Vector calculus is difficult for all students. It combines algebra and geometry in ways that go beyond high school training. Students need frequent and consistent responses. Without assurances they are learning a model supporting the course’s ideas, students revert to memorizing details and superficial imitation of the instructor’s blackboard topics. Many bog down, lose interest, and drop the course.

1.1. A Great Need. I needed more resources to give a class of 65 students the help they were clamoring for. So, I set out to develop a system of e-mail programs that would give more effective interaction with students. Here were my goals:

(1.1) Retain minority students
(1.2) Enhance interaction between professors and students through project and portfolio creation
(1.3) Find out why—despite good evaluations—fundamental topics didn’t work
(1.4) Continue to live a reasonable life while accomplishing (1.1) and (1.2)

1.2. Why Technology. I chose technology because it is the only resource we have more of than five years ago. We have less money and less time. Further, a Unix e-mail account is cheap and powerful, once you learn how to use it. Here are basic elements of the program, which was funded by the Sloan Foundation.

(2.1) Establish quality student/instructor e-mail communication
(2.2) Gather student data through e-mail Interactive Questionnaires (IQs)
(2.3) Develop a system of automated portfolios
(2.4) Create a process for developing individual and team projects
(2.5) Reinforce student initiative toward completion of projects
(2.6) Collect weekly comment files for students and teaching assistants

1.3. Need for an office System. These elements required designing and programming seven modules. Complete documentation will appear in [3]. [2] has more discussion of what to expect from on-screen use of the programs. Each module incrementally installs into existing e-mail Unix accounts. These are now available to others at my institution. For example, having teaching fellows use parts of the system improved their performance and communication with me. Using the system requires training on basics, like how to move to a directory in a typical e-mail account. In this article I concentrate on IQs and automated portfolios.

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The intended reader appreciates how little problem solving guidance students get in class, and how they are on their own to handle giving meaning to learning an overstuffed curriculum.
1.4. What the System Did. This e-mail system allowed efficient fielding of 30–40 quality interchanges a day. Further, it kept formatted electronic portfolios tracking each student’s interactions with me, in several forms.

2. Method

2.1. IQ Evaluation and Modularity. An IQ is an enhanced interactive quiz, actually a computer program, offering various aids and guidance to students who receive it by e-mail. Students take IQs at computer terminals—any terminal with access to their mail accounts. When the student finishes, the IQ automatically returns the student-entered data to the instructor by e-mail for (automatic) placement in the student’s portfolio. The IQ reader software sorts and manipulates evaluative data entered by the student and returned by the IQ.

Teaching the modularity of mathematics is exceptionally hard. IQs’ highly modular structure assists this process. They help students focus on analyzing one step at a time. Data from an IQ is in pieces. You view pieces extracted from a student (or many students’) response(s) to any IQ with the IQ reader. Therefore, IQs help an instructor focus on small conceptual problem elements in a class.

For example, here is a question from vector calculus, divided into parts, to help students analyze lines in 3-space. (Numbers after each part show the point value that part contributes to the 45 points.)

2.2. Comparing lines. 45. Consider two lines

\[ L_1 = \{ (2,1,3) + t(v_x,v_y,v_z) \mid t \in \mathbb{R} \} \quad \text{and} \quad L_2 = \{ (x_0,y_0,z_0) + s(2,1,0) \mid s \in \mathbb{R} \}. \]

You choose the vector \( (v_x,v_y,v_z) \) and the point \( (x_0,y_0,z_0) \) to construct lines with certain properties.

a) 7: For what vectors \( (v_x,v_y,v_z) \) is \( L_1 \parallel L_2 \)?

b) 13: Suppose \( (x_0,y_0,z_0) = (2,1,3) \). Let \( V \) be the set of nonzero vectors \( (v_x,v_y,v_z) \) which make \( L_1 \parallel L_2 \). Find vectors \( v_1, v_2 \in V \) such that \( v_1 \) is perpendicular to \( v_2 \).

c) 10: Suppose \( (v_x,v_y,v_z) = (1,3,2) \), and \( L_1 \) and \( L_2 \) meet at a point. Find a vector \( w \) and a point \( (x_1,y_1,z_1) \) so the plane \( P = \{ (x,y,z) \mid w \cdot ((x,y,z) - (x_1,y_1,z_1)) = 0 \} \) contains all points of both \( L_1 \) and \( L_2 \).

d) 15: Suppose \( (v_x,v_y,v_z) = (1,3,2) \). For what points \( (x_0,y_0,z_0) \) is there a plane \( P \) that contains both \( L_1 \) and \( L_2 \)?

An IQ presents each problem part, with help screens, separately. We (now) use simple \TeX{} notation (see §3.3). Here is an IQ version of problem b above.

b) 13 Let \( (x_0,y_0,z_0) = (2,1,3) \). Let \( V \) be the nonzero vectors \( (v_x,v_y,v_z) \) which make \( L_1 \perp L_2 \). Find two perpendicular vectors \( v_1, v_2 \) in \( V \).
4. 3 Solving equations for $\mathbf{v}_1$ and $\mathbf{v}_2$.

The student enters a menu choice. Choosing 0 lets the student type written answers into the IQ (for return to the instructor). Students can retry menus any time. After returning the IQ to the instructor, a student can go through it again, this time using the help screens to check the work. As a developer of IQs, I work on enhancing the quality of evaluation and interaction.

2.3. Polling Portfolios. To poll a portfolio is to collect IQ and other interaction data from the portfolio. Polling portfolios gives an instructor an instantaneous screen or file report on how a collection of 65 students answered a specific question. Unless I choose otherwise, only I have access to the portfolio.

The phrase automated portfolios refers to creating (additions to) portfolios semiautomatically from interactions. Each problem piece in an IQ has a tag. This simplifies moving material from a student’s IQ responses (e-mail messages), to the student’s portfolio, and gathering the responses of all students to a given question so the instructor can evaluate them. It is as if the instructor had ripped the bluebook apart to allow grading all students’ responses to a give item together. Placing items correctly into the portfolio is what allows easy polling (creation of reports about the portfolio contents). IQs are much easier to grade than similarly enhanced paper and pencil exams. The viewer program for that IQ can create a report across the student portfolio collections using responses to that tagged piece. IQ technology allows the following activities, under the rubric we call polling portfolios.

- Automatic placing/formatting of IQ responses into student portfolios
- Batched evaluating and commenting on specific pieces from an IQ
- Automatic mailings of commented IQ portions to portfolio owners [2]

Here the IQ reader shows answers from students with mail-names akoines and gmoore to problem b (two from 60+), with my responses. Each student with difficulties similar to akoines in the last steps, got the verbal response for akoines. Student responses to problem b showed two types of difficulty among all 60+ answers. Other students tried to start the problem without using the help screens. The response to them showed what they would have gained by doing so.

b) 13 Let $(x_0,y_0,z_0)=(2,1,3)$. Let $V$ be the nonzero vectors $(v_x,v_y,v_z)$ which make $L_1 \perp L_2$. Find two perpendicular vectors $\mathbf{v}_1, \mathbf{v}_2 \in V$.

akoines:-b)- $(x_0,y_0,z_0)=(2,1,3)$, so $L_2=\{(2,1,3)+s(2,1,0) | s \in \mathbb{R}\}$. Help said as $L_1$ and $L_2$ both go through $(2,1,3)$, they are perp. means directions are perp.. So, $(v_x,v_y,v_z)$ is perp. to $(2,1,0)$. Dot product gives $2v_x+v_y=0$. I didn’t get what means the general solution. Help said let $v_z$ be anything, solve the relation between $v_y$ and $v_x$.

RESPONSE (to akoines): List vectors $(v_x,v_y,v_z)$ with $2v_x+v_y=0$. Take $(v_x,v_y,v_z)$ from $V=\{(u,-2u,v) | u,v \in \mathbb{R}\}$ as in b3 help: $u=0$, $v=1$, and $v=0$, $u=1$ as in b4 help give $\mathbf{v}_1=(0,0,1)$ and $\mathbf{v}_2=(1,-2,0)$. [b1.2,b3.3] &&& +8.

gmoore:-b)- As $(x_0,y_0,z_0)=(2,1,3)$, $L_2=\{(2,1,3)+s(2,1,0) \}$. Since $L_1$ and $L_2$ go through $(2,1,3)$, they are perpendicular if their vectors
are perpendicular. The product \((v_x,v_y,v_z).(2,1,0) = 2v_x + v_y\) is 0. So, \(v_z\) is anything, and \(v_x\) and \(v_y\) satisfy this equation.

RESPONSE: Same as to akoines, except this student used different help screens, and so receives a different score.

2.4. Characteristic IQ Phenomena. You see here student use of help screens: the \([b1,2,b3.3]\) in akoines response indicates that akoines used help screens 1 and 3, resulting in a loss of \(2 + 3 = 5\) points from the point value of part b, giving him a score of \(+8\) for that part. The &&& is just a marker showing the end of the response. Solving b) required using the solution of one linear equation in two unknowns. Even with help screens, this was a hard analysis for students. So, the same grader response appeared in many IQs. The viewer lets the grader—seeing many answer parts juxtaposed on the screen prior—simultaneously add this one response to many IQs. This typifies how a batch response to problem pieces cuts grading time. My upcoming book [3] features many applications of the system for batch and personalized merge mail, including returning IQs to students.

3. Findings

Enhanced e-mail interactions brought more contact with students in one course than I had in my previous 20 years of teaching. Without this system, these interactions and associate evaluations would have overwhelmed me.

3.1. Student Perspective. Ten weeks of material is a few succinct ideas. Instructors know where the road goes. Portfolio data shows few students know there is a road. Day to day, students lose their way, and we march on without them. Many faculty are aware students get too little feedback, and that giving more without some aid would overwhelm instructors. I now routinely retain close to 100% of my students. Often students report to me from the next class, on how some particular idea improved their ability to use my course in the next. Here is an example.

\begin{quote}
Dr. Fried, you wouldn’t believe it. I showed the teaching fellow [how to use parametric lines]. The teaching fellow was explaining [curl] and he didn’t understand you had to parametrize the bounding curve. I showed the whole class what parametrization was about. You wouldn’t believe it.
\end{quote}

3.2. Documenting Student Learning. Further, these portfolios started the process of documenting the value added by the instructor. Side messages from students—also put into their portfolios—show specific changes of attitude. Epiphany occurs when students realize it’s not luck if an exam suits their studying. Rather than being lucky to have memorized a problem rubric, they see they need more concentration to maintain an analyzing mode. My web site [2] quotes a student likening her negative approach to analyzing problems with her difficulties using a modem.

3.3. \TeX versus Paper and Pencil. Using \TeX helped my whole class, for the first time ever, use precision in their mathematical writing. Computer shorthand for mathematics formulas (via \TeX) aids effective use of mathematics e-mail. The instructor can teach this incrementally. Further, running e-mail exchanges through a \TeX compiler translates symbols to magnificent output, a reward for students.
4. Use of Findings

Suppose in a particular class I find students show they have forgotten material from two weeks before. I use portfolios to show us all what is happening. I ask the polling program to list questions from previous IQs. When I find one that illustrates (say from an IQ two weeks ago) the same questions students couldn’t answer in today’s class, I create a report on the old responses to that question. That takes no time: I just choose “create a report” from the menu in the program.

4.1. Report Value. The report of students’ responses several weeks ago astounds them. Most would have asserted we never had that material before. It shows them they must consider anew the effectiveness of their study habits, and other aspects of how they respond to classroom data.

Student portfolios contain interactions, including with classmates, which lay out their intellectual problems. Until I used IQs and portfolios, I didn’t realize how often students lose what they have learned. Despite years of excellent interactions with students, by the early 90’s I felt students were deceiving me. They appeared to be masters at convincing me they understood material that in truth, they did not. Using IQs showed what was happening.

4.2. Losing Material. [2] gives detailed screen shots from IQ sessions. A graphic with it illustrates polling portfolios. This shows the dynamics of students learning and losing core material. Students had a tenuous grasp on material soon after a preliminary introduction. Then, weeks later, they lost it. The viewpoint from high school kept reclaiming territory from university ideas. This happened with characteristic material repeatedly (example: falling back on point/slope lines in 3-space). Polling student portfolios simplified catching this and interceding.

4.3. Benefits for the Rest of the Campus. My Sloan Technology activities have enjoyed some campus support (for trying to handle 18,000 students). Steve Franklin and Leonard Meglioli [1] at UCI’s Office of Academic Computing have replicated simple IQ s in HTML forms under the name ZOT dispatch. OAC offers training in it to induce faculty to get students’ comments on their Web offerings. It is a valid, though limited, entrance to the whole portfolio system.

5. Success Factors

Any use of paper, like students responding to IQs on paper rather than on-line, bogged down recording the effort and responding for the teaching fellow or me.

My upcoming book [3] discusses at length why office hours can’t come close to the effectiveness of IQs. Especially: The Sloan system gives an electronic record—put to many uses—and it has all students participate.

References

[3] Interactive E-mail; Enhanced Student Assessment, Book in preparation, 6 of 10 chapters complete as of August 1997.

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