Continuing with the proof of Thm 1

<u>Case 3</u> No cutpoint in $[\rho, On^M)$

<u>Fact</u> For all good P, $P \trianglerighteq M|\rho$

$P \trianglelefteq M \Leftrightarrow \exists E$ in M (not necessarily on the M-sequence) s.t. $crit(E) < \rho$ and E is ρ-strong and $P \triangleleft Ult(M|\kappa^+, E)$

Define N^{+ext} = the stack of all such P where $N = M|\rho$

<u>Proof</u> Farmer's dissertation

<u>Case 4</u> otherwise. ρ singular and there is a cutpoint in (ρ, On^M) but ρ is not a cutpoint.

Say γ is a cutpoint of M in this interval. Then it suffices to identify $M|\gamma$, and then proceed as in Case 2.

<u>Identify $M|\gamma$:</u> Given N a good pm, say N is <u>strong</u> iff N^{+hulls} is well-defined and ~~they~~ the domain of N^{+Hulls} is the same as domain (M) and every proper I.S. of N^{+Hulls} satisfies standard fine-structural condensation.

<u>Claim</u> All strong N are segments of M.

<u>Proof</u> P, Q are strong but not lined up. Define

$P_0 = P \quad Q_0 = Q$

P_{n+1} = the least $P' \trianglelefteq p^+\mathrm{Hulls}$ s.t. $Q_n \in P' \wedge P_n \triangleleft P'$

Q_{n+1} = defined dually

Let P_ω, Q_ω are corresponding stacks. They have the same domains, denote them R.

E^{P_ω} is definable in R from parameter P_0 because we can run the definition of $p^+\mathrm{Hull}$'s inside R. Similarly for E^{Q_ω}. Hence $\mathrm{rud}(P_\omega), \mathrm{rud}(Q_\omega)$ have the same domain and $\Sigma_1^{\mathrm{rud}(P_\omega)}$ is $\Sigma_1^{\mathrm{rud}(Q_\omega)}(\{P_\omega\})$. Then for a cardinal $\eta < \rho$

$$\mathrm{Hull}_{\Sigma_1}^{\mathrm{rud}(P_\omega)}(\eta \cup \{p_1^{\mathrm{rud}(P_\omega)}\}) = \mathrm{Hull}_{\Sigma_1}^{\mathrm{rud}(Q_\omega)}(\eta \cup \{p_1^{\mathrm{rud}(Q_\omega)}\})$$

These collapse to P^*, Q^* are 1-sound; since

$\rho_1(\mathrm{rud}(P_\omega)) = \rho_1(\mathrm{rud}(Q_\omega)) \overset{\mathrm{def}}{=} \rho$.

Take η be large enough s.t. P_ω, Q_ω are in these hulls to make these hulls equal and ~~inch~~ include solidity witnesses to guarantee 1-soundness.

By Σ_1-condensation $P^*, Q^* \trianglelefteq M|\rho$. Now these Hulls see the agreement / disagreement between P_ω, Q_ω.

Thm 2 – Toward the proof

M tame, ω_1 exist $\Rightarrow$ Q^M exists and is M-definable from $x \in \mathbb{R}$.

Claim $\exists \alpha < \omega_1\ \forall P$

P ω-sound, projects to ω and $M|\alpha \trianglelefteq P$ and P is ω_1-iterable above α $\Rightarrow$ $P \triangleleft M|\omega_1$

For $N \triangleleft M|\omega_1$ and P s.t. N, P are ω_1-pm say that (N, P) is bad if $N|\omega_1^P = P|\omega_1^P$ but $N \neq P$ and P is ω_1-iterable above ω_1^P. (Note: N is also ω_1-iterable)

Using Fact Schindler-Steel, we want to compare (N, P). For N as above define a partial ω_1-IS Λ^N for N using methods of Sch-St. Λ^N acts on trees $\mathcal{T}$ s.t.

- $\lambda = lh(\mathcal{T}) < \omega_1$ limit
- $\mathcal{T}$ normal ω-maximal
- $\delta = \delta(\mathcal{T})$. Then $M|\delta$ is $\mathbb{B}_\delta$-generic over $M(\mathcal{T})$. (extender algebra)
- $\mathcal{T}$ is definable over $M|\delta$ from parameters and $\mathcal{T}$ is according to Λ^N so far

Then $\Lambda^N(\mathcal{T})$ = the unique cwf branch b s.t. $Q \stackrel{\text{def}}{=} Q(b, \mathcal{T}) =^* \text{some } Q^* \trianglelefteq M$

This works, as we always have a level of M above $\delta(\mathcal{T})$ which projects to ω.

Compare bad pair (N, P) using (Λ_N, Σ_P).
Here Σ_P is some ω_1-strategy for P.
Describe comparison in stages $\langle \delta_\alpha \rangle_{\alpha < \omega_1}$ continuous, δ_α are exactly the Woodins of $M(\mathcal{T}, \mathcal{U})$ when α is a successor ordinals.

$\delta_0 = 0$. To define $(\mathcal{T}, \mathcal{U}) \restriction \delta_1$

Given $(\mathcal{T}, \mathcal{U}) \restriction (\gamma + 1)$
- identify the least disagreement in extenders
-

$\lambda_1 =$ least λ s.t.

$$Q(\mathcal{T} \restriction \lambda, \Lambda^N(\mathcal{T} \restriction \lambda)) \neq Q(\mathcal{U} \restriction \lambda, \Sigma_P(\mathcal{U} \restriction \lambda))$$

$$\delta_1 = \delta((\mathcal{T}, \mathcal{U}) \restriction \lambda_1) \Rightarrow \delta_1 = \lambda_1$$

Claim $(\mathcal{T}, \mathcal{U}) \restriction \lambda_1$ is well-defined, i.e. $\mathcal{T} \restriction \lambda_1$ is via Λ^N