Continuing with the proof of Theorem 1

Case 3: No cutpoint in $(p, 0, \infty)$

Fact: For all good $P$, $P \not\in M_1 P$

$P \not\in M \Rightarrow E \in M$ (not necessarily on the $M$-sequence) s.t.

$\exists t(E) < p$ and $E$ is $p$-strong

and $P \not\in \text{Ult}(M_{1+p}, E)$

Define $N^\text{ext}$: the stack of all such $P$ where $N = M_1 P$

Proof: Farmer's dissertation

Case 4: otherwise $p$ singular and there is a cutpoint in $(p, 0, \infty)$ but $p$ is not a cutpoint.

Say $x$ is a cutpoint of $M$ in this interval.

Then it suffices to identify $M[x]$, and then proceed as in Case 2.

Identify $M[x]$: Given $N$ a good $pm$, say $N$ is strong iff $N$-hulls is well-defined and the domain of $N$-hulls is the same as domain $(M)$ and every proper I.S. of $N$-hulls satisfies standard fine-structural condensation.
Claim: All strong $N$ are segments of $M$.

Proof: $P, Q$ are strong but not lined up. Define $P_0 = P$, $Q_0 = Q$.

$P_{w+1} =$ the least $P' \subseteq P^+Hull$ s.t. $Q_0 \in P \wedge P \cap P' = \emptyset$. $Q_{w+1} =$ defined dually.

Let $P_w, Q_w$ be corresponding stacks. They have the same domain, denote them $R$.

$E^{P_w}$ is definable in $R$ from parameter $P_0$.

Because we can ran the definition of $P^+Hull$ inside $R$.

Similarly for $E^{Q_w}$. Hence $\text{End}(P_w)$, $\text{End}(Q_w)$ have the same domain and $E^{\text{End}(P_w)}$ is $\Sigma_1$ in $\text{End}(Q_w)$ ($\{P_w/y\}$). Then for a cardinal $\eta < \rho$,

$$\text{Hull}_{\Sigma_1}(\eta \cup \{P, \text{End}(P_w)\}) = \text{Hull}_{\Sigma_1}(\eta \cup \{P, \text{End}(Q_w)\})$$

These collapse to $P^+, Q^+$ are $1$-sound; since

$$P_1(\text{End}(P_w)) = P_1(\text{End}(Q_w)) \equiv \rho.$$

Take $\eta$ be large enough s.t. $P_w, Q_w$ are in these hulls so make these hulls equal and embed include solidity witnesses to $\text{End}(R)$-soundness.

By $\Sigma_1$-condensation $P^+, Q^+ \equiv M \! / \! P$. Now these hulls see the agreement/disagreement between $P_w, Q_w$. 
Thus $\mathcal{M}$ — Toward the proof.

$\mathcal{M}$-finite, $\omega_1$ exists $\Rightarrow \omega_1$ exists and is $\mathcal{M}$-definable from $x \in \mathcal{M}$.

Claim 2: $x < \omega_1 \forall \mathcal{P}$

$P$ $\omega$-sound, projects to $\omega$ and $\mathcal{M} \vdash P$ and $P$ is $\omega_1$-iterable above $x \Rightarrow \mathcal{P} \in \mathcal{M} \upharpoonright \omega_1$

For $\mathcal{N} \models \mathcal{M} \upharpoonright \omega_1$ and $\mathcal{P}$ s.t. $\mathcal{N} \models \mathcal{P}$ are $\omega_1$-perm.
say that $(\mathcal{N}, \mathcal{P})$ is bad if $\mathcal{N} \upharpoonright \omega_1 \Vdash \mathcal{P} \upharpoonright \omega_1$

but $\mathcal{N} \not\models \mathcal{P}$ and $\mathcal{P}$ is $\omega_1$-iterable above $\omega_1$.

(Note: $\mathcal{N}$ is also $\omega_1$-iterable)

Using Fact: Schindler-Steel, we want to compare $(\mathcal{N}, \mathcal{P})$. For $\mathcal{N}$ as above define a partial $\omega_1$-IS $\mathcal{N}^*$ for $\mathcal{N}$ using methods of Sch-St.

$\mathcal{N}$ acts on trees $\mathcal{T}$ s.t.

- $\mathcal{T} = \mathcal{L}^*(\mathcal{F}) < \omega_1$ limit
- $\mathcal{T}$ around $\omega$-maximal
- $\mathcal{T} = \mathcal{F} \upharpoonright \mathcal{T}$, then $\mathcal{M} \upharpoonright \mathcal{T}$ is $\mathcal{B}_1$-generic over $\mathcal{M} \upharpoonright \mathcal{T}$
- $\mathcal{T}$ is definable over $\mathcal{M} \upharpoonright \mathcal{T}$ from parameters

Then $\mathcal{N}^*(\mathcal{T}) = \text{the unique c.w.f. branch of}$

$s.t. \mathcal{Q} \equiv \Phi \langle b, \mathcal{T} \rangle = \langle \varphi \rangle \text{ some } \varphi \in \mathcal{M}$
This works, as we always have a level of \( M \) above \( \delta ( \nu ) \) which projects to \( \nu \).

Compare bad pair \(( \nu , \nu )\) using \(( \lambda, \Sigma )\).

Here \( \Sigma \) is some \( \mu \)-strategy for \( P \).

Describe comparison on stages \( \langle \delta_\alpha \rangle_{\alpha < \chi_1} \) continuing.

\( \delta \) are exactly the Woodin of \( M ( T, \nu ) \) when

\( \chi \) is a successor ordinal.

\( \delta_0 = 0 \). To define \(( T, \nu ) \downarrow \delta_1 \)

Given \(( T, \nu ) \downarrow ( \gamma + 1 )\)

- Identify the least disagreement in extenders

\[ \lambda_1 = \text{least } \lambda \text{ s.t.} \]
\[ Q ( \tau \downarrow \lambda, \Lambda ( \tau ( \lambda ) ) ) \neq Q ( \nu \downarrow \lambda, \Sigma , \nu ( \lambda ) ) \]

\[ \delta_1 = \delta ( \delta ( T, \nu ) \uparrow \chi_1 ) \Rightarrow \delta_1 = \chi_1 \]

Claim \(( T, \nu ) \uparrow \chi_1 \) is well-defined, i.e.

\[ ( T, \nu ) \uparrow \chi_1 \text{ is a } \Lambda \nu \]