Theorem. Suppose \( n \) is supercompact, \( n < \lambda \) where \( \lambda \) is superstrong, and where an arbitrarily large Woodin cardinals. Assume UBH holds for "nice" iteration trees above \( n \). Then

(a) There is an \( \bar{\imath} \) that \((P, \Xi)\) s.t.

\[ P \models \text{Theo is a superstrong cardinal.} \]

(b) There is a pointclass \( \Gamma \subseteq \text{Hec}_n \) s.t.

(i) \( \text{HOD}^{L(n,R)} \models \text{Hec}_n \) is a superstrong

(ii) \( A \in \Gamma \models \text{HOD}^{L(n,R)} \models "I \text{ am a \ellpm}" \)

\( \text{REM: UBH + \( n \) supercompact \( \Rightarrow \) V is uniquely iterable above \( n \) for stacks of normal trees. (Woodin)} \)

\( \text{REM: P as in (a) is given by a \ellpm \ construction on V} \)

\( \text{REM: P in (b) comes from IR - genericity iteration} \)

\( P \rightarrow \mathcal{Q} \) as \( D(\mathcal{Q}, < \lambda) \).

II. Normalizing well

2-step. Given \( M \) an \( \ellpm \), \( \mathcal{U}(F), \langle \lambda_E \rangle \models \text{UBH} \)

\[ \text{Claim: } \frac{\mathcal{U}(F), \langle \lambda_E \rangle \models \text{UBH}}{\mathcal{N}} \]

\[ \begin{array}{c}
\xrightarrow{i_E} \\
\rightarrow \\
F \rightarrow \mathcal{Q} \rightarrow R \\
{\text{4 comes from the shift lemma}}
\end{array} \]

\[ \mathcal{V} \models \lambda_E = i_F^\mathcal{V} \lambda_E \] 

\[ \text{so } F = E_4 \mathcal{H}(F) \]

\[ \text{so: } \mathcal{V} = \sigma \circ i_E \]
Here we avoid dropping cases to keep the complexity low.

For $T = \langle E \rangle, U = \langle F \rangle$:

$W(T, U) = \langle E, F, i^M_F(E) \rangle$ (if $h_U(F) > h_L(E)$)

Have: $\sigma_0: \text{last of } U \rightarrow \text{last of } W(T, U)$

$\sigma_0: \langle T, U \rangle = \langle W(T, U) \rangle$

Case 2: $c_U(E) < c_U(F) < \lambda_E$

\[ \begin{array}{c}
\xymatrix{n & \sigma(T) \ar[r] & T \ar[r] & \langle E, F, i^M_F(E) \rangle \\
M & \ar[r]^{i^M_E(F)} & \ar[r]^{i^M_E(F)} & \end{array} \]

\[ \begin{array}{c}
4 \text{ again comes from the shift lemma.} \\
4(\{a_t^+\}_{t \in E}) = \{i^M_E(F), f\}_t^{i^M_F(E)}
\end{array} \]

$W(\langle E \rangle, \langle F \rangle) = \langle E, F, i^M_F(E) \rangle$

REM: $W$ is called the embedding normalized $\sigma$ of $\langle T, U \rangle$.

There is also a full normalized in $X(T, U)$ with last model $Q$. Here e.g. in Case 1 use $i^N_E(E)$, not $i^M_F(E)$. $i^N_E(E)$ is on the sequence of $P$ by condensation, so this is most useful in developing the theory as it is the end product.

Now given $T$ normal on $M$ and $F$ with $n = c_U(F)$,
Define \( W(T, F) \) for \( F \) on the sequence \( M_{\beta} \); here \( \mathcal{F} \) is normal on \( \mathcal{M} \). Let
\[
\beta \mapsto \beta^* \quad \text{s.t.} \quad \nu < \chi(\mathcal{E}^\beta) = \text{the least } \nu \text{ s.t. } \nu < \chi(\mathcal{E}^\beta)
\]
\[
\delta = \text{least } \delta \text{ s.t. } F \in \mathcal{M}^\delta \text{ on the sequence of } M_{\beta}.
\]

Assume
\[
M^\beta_{\beta} = M^\gamma_{\beta}, \quad \mathcal{T}^\beta_{\beta+1} = \mathcal{Y}^\beta_{\beta+1}
\]
\[
\beta \leq \delta, \quad \mathcal{E}^\beta \text{ exists } \implies \text{dom}(F) \subseteq \chi(\mathcal{E}^\beta). \text{ Then}
\]
\[
W(T, F) = \mathcal{Y}^{(\delta + 1)}(F) \implies \mathcal{F}^\gamma, \quad \gamma \geq \beta
\]

Define:
\[
\phi(\beta) = \begin{cases} 
\mathcal{E}^\beta & \forall \beta < \delta \\
(\delta + 1)^{\gamma-\beta} \quad \forall \beta \leq \delta \leq \Theta
\end{cases}
\]

Here \( \Theta + 1 = \text{lh}(T) \).
Define \( \pi_j : m_j^\sigma \rightarrow m_j^\nu \) as we do.

\( \pi_j = \text{id} \) for \( j < \beta \).

\[ m_{\alpha+1}^\nu = \text{hlt} (m_{\beta}^\nu, \varphi(\beta)) \quad (\text{Non-dropping case}) \]

\( \pi_\beta : m_\beta^\sigma \rightarrow m_\beta^\nu \) canonical embedding.

For \( \gamma \geq \beta \), \( \eta = \text{pred}(\beta+1) \) we let

\[ E_\gamma^\nu = \pi_\beta (E_\gamma^\sigma) \]

(a) \( \alpha (F) \leq \alpha (E_3^\sigma) \)

\[ w \text{-pred} (\varphi(\beta+1)) = \varphi (\text{pred}(\beta+1)) \]

\[ m_{\varphi(\beta+1)}^\nu = \text{hlt} (m_{\varphi(\gamma)}^\nu, E_{\varphi(\gamma)}^\nu) \]

\[ \pi_{\beta+1} : m_{\beta+1}^\sigma \rightarrow m_{\varphi(\beta+1)}^\nu \quad \pi_{\beta+1} [\varphi, t] = [\pi_{\beta+1} \varphi, t_{\beta+1}] \]

(b) \( \alpha (F) > \alpha (E_3^\sigma) \)

Then \( \gamma \leq \beta \). Then \( w \text{-pred} (\varphi(\beta+1)) = \gamma \).

\( \pi_\beta \)'s commute with embeddings of \( T(W) \).

So can define \( \pi_\lambda : m_\lambda^\sigma \rightarrow m_\lambda^\nu \)

\[ \varphi(\lambda) \]
Warning: Not always true that \( g(T - \text{pred}(i+1)) = w - \text{pred}(\text{pred}(i+1)) \).

For \( U \) a normal tree and \( n+1 \leq lH(U) \):

\[ S^U \]

For \( U \) a normal tree and \( n+1 \leq lH(U) \):

\[ S^U \]

\[ \text{Branch extender} \]

\[ \text{For } U \text{ a normal tree and } n+1 \leq lH(U) \text{, the sequence of branch extenders used in } U \text{ getting to } M^U \]

\[ \text{Vert} = \{ \}

\[ \text{C} \]

\[ \text{F}(C) \]

\[ \text{C} \]

\[ \text{F}(U) \]

\[ \text{C} \]

\[ \text{F}(H) \]

\[ \text{C} \]

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\[ \text{F}(H) \]