2016 - 07 - 22 9:30 JOHN STEEL (1)

<u>Theorem</u> Suppose κ is supercompact, $\kappa < \lambda$ where λ is superstrong, and there are arbitrarily large Woodin cardinals. Assume UBH holds for "nice" iteration trees above κ. Then

(a) There is an lbr hod pair (P, Σ) s.t.

$P \models$ there is a superstrong cardinal.

(b) There is a pointclass $\Gamma \subseteq \mathrm{Hom}_\infty$ s.t.

(i) $\mathrm{HOD}^{L(\Gamma, \mathbb{R})} \models$ there is a superstrong

(ii) $\forall \Gamma' \subseteq \Gamma$: $\mathrm{HOD}^{L(\Gamma', \mathbb{R})} \models$ "I am a lpm"

<u>REM</u> UBH + κ supercompact $\Rightarrow$ V is uniquely iterable above κ for stacks of normal trees. (Woodin)

<u>REM</u> P as in (a) is given by a lpm construction in V

<u>REM</u> Γ in (b) comes from $\mathbb{R}$-genericity iteration $P \longrightarrow Q$ as $D(Q, < \lambda)$.

<u>II Normalizing well</u>

<u>2-step</u> Given M an pm, $cr(F) < \lambda_E$, ~~(...)~~

<u>Case 1:</u>
$\alpha(F) \le \alpha(E)$

$$\begin{tikzcd}
N \arrow[r, "i^N_F"] \arrow[rr, bend left, "\psi"] & Q \arrow[r, dashed, "\sigma"] & R \\
M \arrow[u, "i^M_E"] \arrow[r, "i^M_F"'] & P \arrow[ur, "i^M_F(E)"'] &
\end{tikzcd}$$

ψ comes from the shift lemma

$\psi \restriction \lambda_E = i^M_F \restriction \lambda_E$ so $F = E_\psi \restriction lh(F)$

so: $\psi = \sigma \circ i^N_F$

Here we avoid dropping cases to keep the complexity low.

~~Some~~ For $\mathcal{T} = \langle E \rangle$, $\mathcal{U} = \langle F \rangle$:

$W(\mathcal{T}, \mathcal{U}) = \langle E, F, i^M_F(E) \rangle$ (if $lh(F) > lh(E)$)

Have: σ : last of $\mathcal{U}$ $\longrightarrow$ last of $W(\mathcal{T}, \mathcal{U})$

$\sigma \circ i^{\langle \mathcal{T}, \mathcal{U} \rangle} = i^{W(\mathcal{T}, \mathcal{U})}$

<u>Case 2</u> $\operatorname{cr}(E) < \operatorname{cr}(F) < \lambda_E$

$$\begin{tikzcd}
N \arrow[r, "i^N_F"] \arrow[rr, bend left, "\psi"] & Q \arrow[r, "\tau"] & R \\
M \arrow[u, "i^M_E"] \arrow[urr, "i^M_F(E)"] \arrow[r, "i^M_F"'] & P &
\end{tikzcd}$$

ψ again comes from the shift lemma.

$\psi([a, f]^M_E) = [i^M_F(a), f]_{i^M_F(E)}$

$W(\langle E \rangle, \langle F \rangle) = \langle E, F, i^M_F(E) \rangle$

<u>REM</u> W is called the <u>embedding normalization</u> of $\langle \mathcal{T}, \mathcal{U} \rangle$.

There is also a <u>full normalization</u> $X(\mathcal{T}, \mathcal{U})$ with last model Q. Here e.g. in Case 1 use $i^N_F(E)$, not $i^M_F(E)$. $i^N_F(E)$ is on the sequence of P by condensation, so this is not useful in developing the theory as it is the end product.

Now given $\mathcal{T}$ normal on M and

F with $\kappa = \operatorname{cr}(F)$, Define $W(\mathcal{T},F)$ → Normal on M for F on the sequence $M^{\mathcal{T}}_\alpha$; here $\mathcal{U}$ is normal on M. Let

$$\beta^{\mathcal{T},F} = \text{least } \beta \text{ s.t. } \kappa < \lambda(E^{\mathcal{T}}_\beta) = \text{the least } \beta \text{ s.t. } \kappa < \lambda(E^{\mathcal{U}}_\beta)$$

$$\alpha = \text{least s.t. } F \text{ is on the sequence of } M^{\mathcal{U}}_\alpha$$

assume

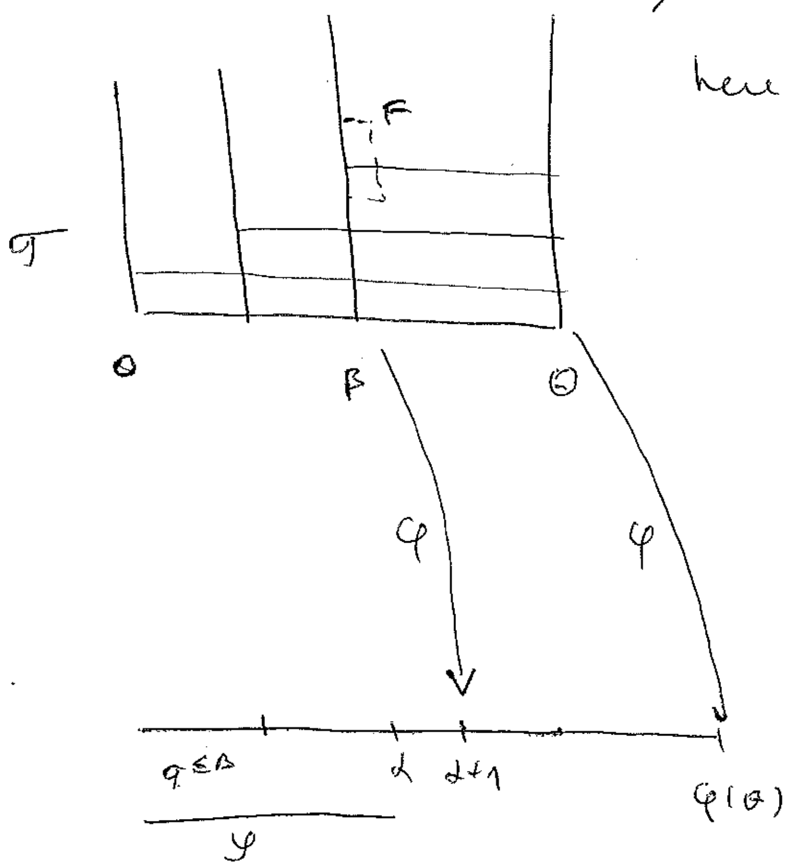
$$M^{\mathcal{T}}_\beta = M^{\mathcal{U}}_\beta, \quad \mathcal{T}\restriction\beta+1 = \mathcal{U}\restriction\beta+1$$

$$\beta \le \alpha, \ E^{\mathcal{T}}_\beta \text{ exists} \Rightarrow \operatorname{dom}(F) \subseteq \lambda(E^{\mathcal{T}}_\beta). \text{ Then}$$

$$W(\mathcal{T},F) = \mathcal{U}\restriction(\alpha+1)^\frown\langle F\rangle^\frown i_F{}''\,\mathcal{T}^{\ge\beta}$$

<u>Define</u>:
$$\varphi(\xi) = \begin{cases} \xi & \text{if } \xi < \beta \\ (\alpha+1)+(\xi-\beta) & \text{if } \beta \le \xi \le \Theta \end{cases}$$

here $\Theta + 1 = lh(\mathcal{T})$



Define $\pi_\xi : M^{\mathcal{T}}_\xi \longrightarrow M^{\mathcal{W}}_{\varphi(\xi)}$ as we go

$\pi_\xi = \mathrm{id}$ for $\xi < \beta$

$M^{\mathcal{W}}_{\alpha+1} = M^{\mathcal{W}}_{\varphi(\beta)} = \mathrm{Ult}(M^{\mathcal{T}}_\beta, F)$ (Non-dropping case)

$\pi_\beta : M^{\mathcal{T}}_\beta \longrightarrow M^{\mathcal{W}}_{\varphi(\beta)}$ canonical embedding.

For $\xi \geq \beta$, $\eta = \mathcal{T}\text{-pred}(\xi+1)$ we let

$E^{\mathcal{W}}_{\varphi(\xi)} = \pi_\xi(E^{\mathcal{T}}_\xi)$

(a) $\mathrm{cr}(F) \leq \mathrm{cr}(E^{\mathcal{T}}_\xi)$ $\quad \xi$

$\mathcal{W}\text{-pred}(\varphi(\xi+1)) = \varphi(\mathcal{T}\text{-pred}(\xi+1))$

$M^{\mathcal{W}}_{\varphi(\xi+1)} = \mathrm{Ult}(M^{\mathcal{W}}_{\varphi(\eta)}, E^{\mathcal{W}}_{\varphi(\xi)})$

$\pi_{\xi+1} : M^{\mathcal{T}}_{\xi+1} \longrightarrow M^{\mathcal{W}}_{\varphi(\xi+1)}$ $\quad \pi_{\xi+1}[a,f] = [\pi_\xi(a), \pi_\eta(f)]$

(b) $\mathrm{cr}(F) > \mathrm{cr}(E^{\mathcal{T}}_\xi)$

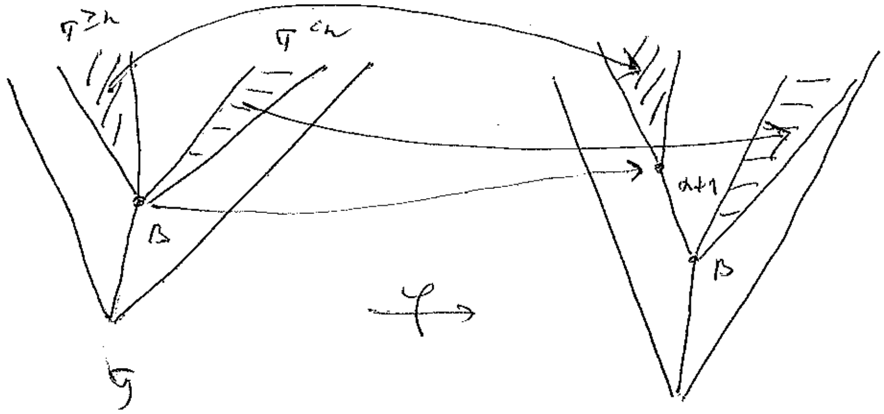
Then $\eta \leq \beta$. Then $\mathcal{W}\text{-pred}(\varphi(\xi+1)) = \eta$.

π's commute with embeddings of $\mathcal{T}, \mathcal{W}$

So can define $\pi_\lambda : M^{\mathcal{T}}_\lambda \longrightarrow M^{\mathcal{W}}_{\varphi(\lambda)}$

<u>Warning</u> Not always true that $\varphi(T\text{-pred}(\xi+1)) =$

$= W\text{-pred}(\varphi(\xi+1))$.



<u>Branch extenders</u> For U a normal tree and $\alpha+1 \leq lh(U)$:

S^U_α = the sequence of extenders ~~...~~ used in U getting to M^U_α

$U^{ext} = \{$