1 pm: Lensing indexing

- add "least missing branch" at appropriate stage; this actually means the branch for the least tree with no branch so far.

- sim type of lpm $M : E^H \models H \models \exists^H \forall^H$  

  branch

  under leaf strategy

  formula under strategy

  merged

- standard finite structure notation. The only difference is adding a parameter $(k(H))$, the degree of soundness. $M$ is $k(H)$-sound.

$M \vdash_{(k)} \phi \iff M \models \forall \phi \wedge M \models \exists \phi \wedge k \leq \phi$.

- $M$-tree is a $\langle \chi_k, T \rangle$ s.t. $T$ is weakly normal on $M \models_{(k)}$.

  weakly normal = length increasing + non-ownlapping

  Dropping: need not drop to the longest initial segment.

- $M$-stack $S = \langle \tau \vdash \eta \cdot \xi \vdash \zeta \rangle \mid \xi \leq \zeta$

- A complete strategy will scope for $M$ is a strategy acting on all $M$-stacks in $\{H \}$. 

- Given a strategy $\Omega$ for $M$ and an $M$-stack $S$, by $\Omega$ the $\Omega_S(\tau) = \Omega(\sigma^\tau)$

  $\Omega_S(\tau_{i, k}) = \Omega(\sigma^n \chi_{i, k} \cdot \eta)$
\[ \mathbb{P}_x, N \implies N = M_{\Phi}(s) | <v, k> \]
\[ \mathbb{P}_x = \mathbb{P}_s, N \] for \( N \not\equiv M \)

- Normalizing well
- Strong null condensation

\[ \text{Det } \Phi \text{ is strategy coherent of } \]
\[ \mathbb{P}_x, s^{-1} = \mathbb{P}_x (<v_0, <f_{\phi}^\Pi>, <v_{s-1}, \theta>) \]

\[ \text{Det } \text{Given } \pi : N \to N \{ <v, k> \} \text{ and } \Phi \text{ a coupled } \Phi, \]

for \( N \) :
\[ \mathbb{P}_x, \pi_{\phi, k} = \pi^- \text{ pullback of } \mathbb{P}_x \]

\[ \text{Det } \Phi \text{ is self-consistent of } \]

(a) whenever \( <v, k> \subseteq <y, l> \subseteq <x, (y, l) \}
\[ \mathbb{P}_x, v_{\phi, k} = \mathbb{P}_x (v, k) \]

(b) the same is true for all tails \( \mathbb{P}_x \)

\[ \text{Det } (M, \Phi) \text{ is a bbr had-pair with scope } \Phi \text{ of } \]

1. \( M \) is a bbr
2. \( \Phi \) is a complete strategy for \( M \) with scope \( \Phi \)
3. \( \Phi \) is normalizing \( \Phi, \not\equiv \Phi \) which has strong
   null condensation, \( \Phi \) is self-strategy coherent and self-consistent
(1) For any $s$ by $\mathcal{R}$, $N \not\subseteq M_a(s)$, we have:

$$\mathcal{R}^N \subseteq \mathcal{R}_{s,N}$$

we say that $(N, \mathcal{R})$ is self-aware

**Remark:** Typically: $V = \mathcal{AD}^+$, $\mathcal{M}$ is ctfb, $\text{scope}(\mathcal{R}) = \mathcal{H}$.

In this situation we get:

1. $\mathcal{R}$ is pullback-consistent: if $\pi: M \to N$ is an inclusion map by $\mathcal{R}$, then

$$\mathcal{R}_{\pi(M), \pi^*N} = \mathcal{R}_N$$

2. $\mathcal{R}$ is positional: For $s, t$ by $\mathcal{R}$, $s \perp t$

$$N \not\subseteq M_a(s) \wedge N \not\subseteq M_a(t)$$

we have $\mathcal{R}_{s,N} = \mathcal{R}_{t,N}$

An l.p.m. construction: Choose $\mathcal{M}_{\mathcal{R}_H}^R$ $M_{\mathcal{R}_H}$ $\mathcal{H}$ $\mathcal{R}_H^C$ $\mathcal{M}_{\mathcal{R}_H}^R$ $\mathcal{R}_H^C$

(A given $\mathcal{R}$ for $V$ assume $V$ is uniquely iterable for normal trees (nice ones). Hence strategy extends to stacks.

$$(M_{\mathcal{R}_H}^C, \mathcal{R}_H^C)$$ are l.o.c. head pair.

Also let $F^H$ are the background extenders by $\mathcal{M}_{\mathcal{R}_H}$ of exist.
\( (1) \) \text{ Granted } (M_{\nu,1}^c, \mathcal{D}_{\nu,1}^c) \text{ exists:}

\( (\iota) \) \( p (M_{\nu,1}) \) is solid

\( (\iota\iota) \) For \( p = p(M_{\nu,1}) \), \( M_{\nu,1}^c (p+M_{\nu,1}) \in \text{ hull}_{\nu,1}^{\mathcal{D}_{\nu,1}^c} \)

i.e. \( M_{\nu,1}^c \cap \text{ conv}(M_{\nu,1}^c) \in \mathcal{D}_{\nu,1}^c \) exists; i.e. among these,
\( p(M_{\nu,1}) \) is minimal

\( (2) \) \( \forall_{\nu,} \) \text{ if } \text{ F}^* \subseteq G^* \text{ can serve as background for }

\( (M_{\nu,1}^c, F) \subseteq (M_{\nu,1}^c, G) \text{ then } F = G \)

\( (4) \) \( \nu, 1 \) is done by induction on \( \nu, \kappa \)

\textbf{Comparison lemma:} Let \( \kappa \) be Woodin and assume \( \nu \in \kappa \) is uniquely minimally iterable for some trees in \( V_\kappa \).

Let \( (P, \Sigma) \) be the good path \( P \in V_\kappa \) and Code \( \langle\Sigma\rangle \) is \( \mathcal{S}-\mathsf{UB} \).

\( P \in \mathcal{H}(\kappa) \). Let \( C \) be an lpm construction with \( F^* \) in \( V_\kappa \),

a "maximal model", that is, \( F^* \) to whenever possible,

then there is \( (\nu, k) \) s.t.

\( (1) \) \( (P, \Sigma) \) satisfies to \((M_{\nu,k}^c, \mathcal{D}_{\nu,k}^c)\)

\( (2) \) \( (P, \Sigma) \) satisfies strictly past \((M_{\nu,k}, \mathcal{D}_{\nu,k})\)

where \( (\nu, k) \) \( \leq_{\text{ext}} (\nu, k) \)
Dodd--Tennenbaum Lemma: For the initial path \( (M, J) \) and a stack \( s \) by \( J \), \( N \in M_{q}(s) \) and \( \pi : M \to N \), s.t.
\[
\overline{J}^{\pi}_{s,N} = J.
\]
Then \( M \to N \) does not drop, so we have \( \iota : M \to N \) an inclusion map and
\[
(\gamma) \leq (\pi, \gamma) \quad \text{f.o. } \gamma \in M \backslash \text{On}.
\]
(Note: The assumption \( \pi \) can be briefly written as: \( \pi : (M, J) \to (N, J_{s,N}) \).)

Corollary: If \( \pi \) itself is an inclusion map by \( J \), then \( (M, J) \to (N, J_{s,N}) \) does not say the stack \( s \) \( \leq \) \( s \) \( \in M_{q}(s) \) and \( J_{s,N} = \overline{J}^{\pi}_{s,N} \) (by pullback consistency)

Thus \( \pi = \iota \).

Corollary: For \( (P, \Sigma) \) an \( lbm \) level pair with scope \( HC \).

Recall: Strategies are automatically both Suslin--co-Suslin; this follows from strong will condensation.

\[
M_{q}(P, \Sigma) = \text{dir.-lim. of all } E\text{-iterated of } (P, \Sigma)_{resb}
\]

Remark: Each \( M_{q}(P, \Sigma) \) is OD. Hence
\[
(P, \Sigma) \subseteq (q, \Pi)
\]

Have a common subtree \( \leq^{k} \) is a PVO by D-J.