Weak DJ property for $(M, \langle \psi \rangle)$: $M = \{ \varepsilon. \langle \varepsilon \rangle. \langle \varepsilon \rangle \}$, get in the DJ situation: $N \rightarrow_{n} N$ does not drop, i.e., $\exists k, n$ s.t. $\langle \psi \rangle \neq \psi_{k}$. Then the least such $n$ is $\langle \psi \rangle(k, \langle \psi \rangle_{n})$.

Thus (Existence of core): Let $M$ be a countable Lpm, $\exists$ a complete strategy for $M$ defined on all countable $\text{stack}_{R}^{L}$, cad model $N$, $(N, \langle \psi \rangle_{N})$ is a lbe had pair. Suppose $\not \models \mathcal{L}$ is valid by a set of rules that is $\text{stack}_{R}^{L}$-valid in some $\mathcal{L}^{R}$. Then $\text{core}(M, \langle \psi \rangle)$ exists:

1. $\not \models \mathcal{L}$ is valid
2. $M \not \models M = \text{Hull}_{M}^{\mathcal{L}}(\mathcal{L} \cup \mathcal{M}(M))$ for $\mathcal{L} = \mathcal{L}(M)$.

Proof: By the usual usual DJ strategy proof: WHTA $M = \{ \varepsilon. \langle \varepsilon \rangle. \langle \varepsilon \rangle \}$ and $\not \models \mathcal{L}$ has the weak DJ property relative to $\mathcal{L}$. Let $r = \mathcal{M}(M)$.

Assume $r = \langle \varepsilon_{1}, \ldots, \varepsilon_{k} \rangle$ some $k$, $\varepsilon_{0} > \varepsilon_{1} > \cdots > \varepsilon_{k}$. Let $r = q \not \models \psi$ when $\psi$ is the longest solid initial segment of $r$. $\text{Max}(s) < \min(s)$. Assume $q \not \models \psi$ and get a contradiction. Similar for universality. Let $\sigma_{0} = \text{least } \beta \text{ s.t. } \text{Th}_{M}^{+}(\beta \cup q) \in M$.

WHEA $\sigma_{0} \in M$. Let $K = \text{the collapse of } \text{Hull}_{M}^{\mathcal{L}}(\sigma_{0}, q)$ $\tau : N \rightarrow M$, $\not \models \sigma_{0} \in K \in M$. 
Claim 0(a) if \( q = \emptyset \), then \( P = \emptyset \).
(b) if \( q \neq \emptyset \) then \( P \leq \bigcup \mathcal{E} \).
(c) \( P \leq \delta \) is a cardinal.

Proof strategy: Compare \((M_1, \mathcal{C}_1, \delta_1)\) vs \(M_1\).

Use \(r(\mathcal{C}_1, \delta_1)\) to dictate \((M_1, \mathcal{C}_1, \delta_1)\), letting \(P : \mathcal{C}_1\) to get \(U\) on \(M_1\), on the other side, \(P\) is a \(M_1\)-side of the comparison. Now we want \(D\) to conclude that the last model \(P\) on the phenoax side is above \(K_i\) and \(P \subseteq \mathcal{C}_1 = M_i^{\delta_1}\). This is in the \(L(\mathcal{C}_1)\).

In the current case, we are comparing against the background.

Problem: When \( P = \mathcal{C}_1 \), \( P \) above \( M_1 \),

\( \cdot \cdot \cdot \rightarrow P \quad \cdot \cdot \cdot \rightarrow \mathcal{C}_1 \quad \text{and} \quad \mathcal{C}_1 \leftrightarrow P \) we got.

1st extension used and \( \cdot \cdot \cdot \rightarrow \mathcal{C}_1 \) are compatible. This gives contradiction in the \( L(\mathcal{C}_1) \) case but not in \( \mathcal{C}_1 \) case, as they may have been hit because of disagreement with \( M_1 \) and not \( \mathcal{C}_1 \) or \( P \).

Let \( N^* \) be a coarse \( \mathcal{F} \)-Woodin model in a model \( L(\mathcal{P}(\mathcal{R})) \) s.t. \( \mathcal{C}_1 \) captures \( \mathcal{F} \). \( M \) is \( \mathcal{C}_1 \)-closed in \( N^* \) and \( N^* \) has a \( \mathcal{B} \) code for \( \mathcal{F} \). The \( \mathcal{B} \) for \( N^* \) enure the \( \mathcal{B} \) codes of \( \mathcal{F} \) to \( \mathcal{B} \) codes of \( P \).

Let \( C \) be a maximal \( \mathcal{B} \)-maximal \( \mathcal{B} \)-construction of \( N^* \).
At work, best. $(M, \delta)$ elements to $(M^C_{\omega_0, \kappa}, D^C_{\omega_1, \varphi})$

the strict part each

$(M, \delta, D^C_{\omega_1, \varphi})$ wherein $(\gamma, \varepsilon) \leq \varepsilon (\varepsilon, \kappa_0)$

have $U_{\gamma, \varepsilon}$ for $(\gamma, \varepsilon)\leq (\varepsilon, \kappa_0)$ on $(M, \delta)$

witnessing this.

We now define by induction on pairs $(\varepsilon, \gamma) \leq (\lambda, \kappa_0)$

maximal trees $Y_{\gamma, \varepsilon}$ on $(M, K_{\omega_0})$ lifted to $T_{\gamma, \varepsilon}$ on $M$

with $(\lambda, \sigma)$. As we go in defining $T_{\gamma, \varepsilon}$

we look at $U_{\gamma, \varepsilon}$ and based on that declare certain

models unstable. All exit extended from $Y$ come

from stable nodes. $Y_{\gamma, \varepsilon}$ has a base model $M_{\gamma, \varepsilon}$

and it's stable $Y_{\gamma, \varepsilon}$ has $\leq 2$ more models than $Y_{\gamma, \varepsilon}$

Also define $\gamma_\theta$ for $\theta < \text{lh}(\gamma)$. Also define $T_{\theta} : M^\gamma_\theta \rightarrow N^\gamma_\theta$

$\theta \leq \sigma \rightarrow \gamma_\theta \leq \gamma_\sigma$.

For $\gamma, \varepsilon$, let $U = U_{\gamma, \varepsilon}$.

$Y = Y_{\gamma, \varepsilon}$ is defined by

- $M^\gamma_\varepsilon = M$
- $H^\gamma_\varepsilon = K$
- $\gamma_\varepsilon = \varepsilon_0$
- $M^\gamma_\varepsilon = H^\gamma_\varepsilon = M$
- $\delta_0 = \omega_0$
- $\pi_\varepsilon = \pi$
- $0$ is unstable, $Y$ is stable.

We will have induction hypothesis $Y$ is unstable.

(i) $0 \leq \gamma \theta$ and $(\delta_1, \theta)\gamma_\theta$ does not drop

$\text{Def.}$ For $\theta$ unstable: $\gamma_\theta = \text{sup } \gamma_\delta [\delta_0]_\theta$
and hence if \( \Theta \) is unstable then

(i) 

(ii) \( x^y_o \leq x_\Theta \leq \rho_k(M^y_o) \)

(iii) every \( \pi \leq \Theta \) is unstable

(iv) \( \exists \gamma \ M^y_o = M^y_\gamma \)

(v) \( c(M^y_o) = \sup_{\Theta, \pi} \{ y \} \)

(vi) \( \sigma_\Theta = \max \beta \) s.t. \( T_{k+1}^{M^y_\beta}(\beta, \pi_\Theta) \notin M^y_o \)

At any stage in the definition of \( Y_{l,e} \) we have some left model \( M^y_o \) \( \gamma \) stable.

The construction of \( Y_{l,e} \) can terminate in one of the two ways:

(1) We reach a stable \( \Theta \) s.t. either

(a) \( M_{r,e} \leq M^y_o \) (and \( \rho_{r,e} = \sum_{\Theta, M_{r,e}} \))

(b) \( M^y_o \leq M_{r,e} \) and \( [\text{root}(\Theta), \Theta] \) does not drop in model or degree

This is a successful comparison of \( (M_{r,e}, \omega_o) \) with \( M \).

(2) We reach a stable \( \Theta \) s.t. for some \( \gamma \)

\( M^y_o = M^y_\gamma \) and neither \( \exists \Theta \ [\text{root}(\Theta), \Theta] \) nor \( G(\Theta)_e \) has dropped. Moreover, \( \xi = M^y_o \langle \delta(M^y_o)_o, -1 \rangle \) we have: \( \xi \leq M_{r,e} \).

(2) is considered a successful comparison.
If $\delta = \emptyset$ does not work as in (1) or (2), then we define $\tilde{M}_y^{y_1}$ and maybe $\tilde{M}_z^{y_2}$.

$\tilde{X}^y$ is the first extender on the sequence of $M^y_{\omega_1}$.

Need to prove: $M_{\omega_1 \varepsilon 1}^{y_1}$ is passive

\[ (\tilde{Z}^y)_{\omega_1} = (\tilde{X}^{\omega_1 \varepsilon 1})_{\omega_1} \]