Connection

Regarding Farley’s Question

Can $E_\theta^y$ applied to $M^y_\omega$ for $\Theta$ unstable.

What if you have to drop?  $\Theta_i$: $\gamma + 1$ is stable

(4) Ralf’s question: Already corrected in the notes.

Theorem. Assume $AD^+$, Let $(M, \mathcal{E})$ be an Hbr hod pair

with scope $HC$. Let code $(\mathcal{E})$ be Sushin-coin-Sushin.

Suppose $M^\mathcal{E} \times M^\mathcal{E}$ is a kind of Woodin + ZFC

Let $\gamma$ be $\text{Cell}(\omega_1, \omega_1)$-generic / $M$, $1^\mathcal{E}_\gamma = \bigcup_\alpha \mathcal{M}^\mathcal{E}(\text{gq}_\alpha)$

and $\text{Hom}_\gamma^x = \{ f : 1^\mathcal{E}_\gamma \to 2^{\mathcal{M}^\mathcal{E}(\text{gq}_\alpha)} \mid f \text{ is } \gamma \text{-s. c.c.} \} $.

Then $\text{Hom}(\mathcal{R}^x_\gamma, \text{Hom}_\gamma^x)$ is an lpm

REM $\frac{\text{L}(\mathcal{R}^x_\gamma, \text{Hom}_\gamma^x)}{\text{AD}_\mathcal{R}^x}$

This follows that the restrictions of strategies $\mathcal{R}^x_\gamma$ are $\mathcal{W}$-Wadge reducible in $\Theta$ (Thus $\Theta$ is that all sets are Sushin).

REM $\frac{\text{L}(\mathcal{R}^x_\gamma, \text{Hom}_\gamma^x)}{\text{AD}_\mathcal{R}^x}$

is an lpm of $M\times \mathcal{E}$.

Q: What is the defining counterpart of such a hod mouse?

Proof: We use the following:

$\text{Theorem}. \text{Assume } AD^+. \text{ Let } (\mathcal{P}, \mathcal{E}) \text{ be a RRb hod pair}

\text{with scope } HC, \text{ and let } \mathcal{E} \subseteq \mathcal{E} \text{ else assume}$

(a) Code $(\mathcal{E}_\mathcal{E})$, $\Sigma$ Code $(\mathcal{E}_\mathcal{E})$ are

$\omega$-Sushin where $\omega = \mathcal{M}_{\omega_1}(\mathcal{P}, \mathcal{E}) \ominus$

(b) Code $(\mathcal{E}_\mathcal{E})$ is $\omega$-Sushin for any $\alpha \in \mathcal{E}$
(1) above follows by Kunen-Martin Theorem.

Regard (ca) above. (idea): if \((P\in\mathbb{E})\) has b.o. \(\mathcal{T}^b\) is a normal manifold dropping tree by \(\mathbb{E}\) if

\[\exists \sigma: M^b_\mathbb{E} \rightarrow M_\mathbb{E}(P\in\mathbb{E})\]

s.t. letting \(\pi^b_{\mathbb{E}}: P \rightarrow M_\mathbb{E}(P\in\mathbb{E})\) be the direct limit map, \(\pi^b_{\mathbb{E}} = \sigma \circ i^b_{\mathbb{E}}\)

In the general case then is a normal tree \(U\) on \(P\) that is by \(\mathbb{E}\) with least model \(M_\mathbb{E}(P\in\mathbb{E})\).

(We have a stack \(<Y_i: i<\omega>\) by \(\mathbb{E}\) with least model \(M_\mathbb{E}(P\in\mathbb{E})\)) Can fully normalise this stack. Yields a \(\mathbb{E}\) simple normal \(U\) on \(P\) with least model \(M_\mathbb{E}(P\in\mathbb{E})\), \(U\) is by \(\mathbb{E}\) in that all its countable elementary submodels are by \(\mathbb{E}\).

Given \(\mathcal{T}^b\) by \(\mathbb{E}\) countable, \(b\) man-dropping: search for a "weak hull embedding" \(\mathcal{T}^b\) from \(\mathcal{T}^b\) into \(U\).

(1) \(\mathcal{T}^b\) is by \(\mathbb{E}\) off \(\exists\) such \(U\)

\[\Leftrightarrow \text{in (4) follows by strong hull condensation}\]

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This is a comparison argument similar to that of the UBH proof. Use Dodd-Jensen property in place of pointwise definability. See "local HOD computation." 

\[ \Rightarrow \text{in } (\kappa) : \text{if } \kappa \ni \delta \exists \mathcal{E} \text{ we have} \]

\[ P \xrightarrow{\top} \mathcal{M}_b^\mathcal{C} \rightarrow \cdots \rightarrow \mathcal{M}_\omega (P, \mathcal{E}) \]

Full normalization gives a single normal \( W \) on \( \mathcal{M}_b^\mathcal{C} \) with least model \( \mathcal{M}_\omega (P, \mathcal{E}) \).

Then normalize the stack \( \sigma W \), denote it \( \mathcal{X}(P, W) \) (full normalisation).

Hence there is a meek hull embedding from \( P b \) into \( U \) (this is what full normalization gives this).

Now back to HOD computation. Recall \( M \models \kappa \) is a limit of Woodin \( \mathcal{R}_i \), \( \text{Huge} \) from \( D (M, \kappa) \).

**Lemma** For each \( \gamma < \lambda \), \( \kappa \in \mathcal{W} \)

\[ \mathfrak{q}^\mathcal{C} \triangleq \mathfrak{q}^{< \kappa \lambda} \cap (\mathcal{H} \text{C}^\mathcal{C})^\mathcal{K} \]

is in \( \text{Ham}^\mathcal{C} \).

**Proof** Use generic indiscernibility. The term gives \( \mathcal{U} \) is representation.
Lemma. These strategies are Wadge cofinal on \( \text{Ham}^* \).

This is because any such set can be reduced to one of these strategies.

Let \( T \subseteq \mathcal{T} \) be a.c.i., in \( \mathcal{M}_2[\mathcal{Z}] \). The question whether an \( \text{Ord} \) colored and comonotonic image of \( T \) is \( \mathcal{T} \)-co-Wadge decides if \( a \) is green or red in our set.

Working in \( L(\mathcal{R}^*, \text{Ham}^*) \), say \( (\mathcal{R}, \mathcal{E}) \) is an \( \text{Ord} \) colored and comonotonic image of \( T \) with

\( (\mathcal{R}, \mathcal{E}) \text{ is \( \mathcal{T} \)-co-Wadge} \)

(a) \( \mathcal{P} \subseteq \mathcal{K}(-\mathcal{E}) \), and has a largest cardinal \( \mathcal{S} \); \( k(\mathcal{R}) = 0 \)

(b) there is a \( \mathcal{P} \)-starred by \( \mathcal{E} \) with

\( \mathcal{M}_{\mathcal{S}}(\mathcal{S}) = \mathcal{E} \), \( \mathcal{P} \)-to-\( \mathcal{E} \) does not chop

and \( \mathcal{S} \) is \( \mathcal{P} \to \mathcal{E} \) is the iteration map then

there is no \( \text{Ord} \) colored and comonotonic image \( (\mathcal{R}, \mathcal{E}) \) with

\( (\mathcal{R}, \mathcal{E}) \text{ is \( \mathcal{T} \)-co-Wadge} \)

and \( \mathcal{P}(\mathcal{R}) \subseteq \mathcal{E} \) and \( \mathcal{F} \mathcal{E} \subseteq \mathcal{E} \).

Lemma. Let \( \gamma \) be a successor cardinal of \( \mathcal{S} \) then

\( (\mathcal{M}_{\mathcal{S}}(\mathcal{S}), \mathcal{F} \mathcal{E}, \mathcal{E}) \) is a full \( \text{Ord} \) colored and comonotonic image of \( \mathcal{S} \) in \( L(\mathcal{R}^*, \text{Ham}^*) \).

(Any \( \text{Ord} \) colored and comonotonic is \( \text{Ord} \), as it is unique by standard arguments - this uses \( \mathcal{Q} \) as a cutpoint.)

In \( L(\mathcal{R}^*, \text{Ham}^*) \) define

\( \mathcal{F} = \{ (\mathcal{R}, \mathcal{E}) \mid (\mathcal{R}, \mathcal{E}) \text{ is a full \( \text{Ord} \) colored and comonotonic} \} \)
$v$ on $\mathfrak{f}$ is defined by

$$(p, \mathfrak{f}) \in (x, \mathfrak{f}) \iff$$

$$\exists (p, \mathfrak{f}) \in \mathfrak{f} \text{ s.t. } (R, \mathfrak{f}) \leq \text{cut}_+ (x, \mathfrak{f})$$

and $(p, \mathfrak{f})$ iterates to $(R, \mathfrak{f})$ without dropping.

$$M_\omega = \text{dir lim } M_\alpha$$

$$\text{Lemma } M_\theta \subseteq \text{HOD}_{/\theta}$$

$$M_\alpha = \bigcup \{ M_\alpha (p, \mathfrak{f}) \mid (p, \mathfrak{f}) \in \mathfrak{f} \}$$

This is clear.

$$\text{Lemma } \text{HOD}_{/\theta} \subseteq M_\alpha$$

For standard argument. (Sketch)

Let $A \subseteq \alpha$, $\beta \subseteq \theta$.

Then $\beta \cap A = \bigcup \{ \beta (\mathfrak{f}_k, \text{Hod}_k) \mid (\beta, \mathfrak{f}_k) \in (\beta, \mathfrak{f}_k) \}$

Can find $(\mathfrak{f}_0, \mathfrak{f}_\alpha) = (\alpha, \mathfrak{f}_\alpha, \mathfrak{f}_\alpha, \mathfrak{f}_\alpha) $ in $M_\theta$

with $\beta (\mathfrak{f}_0, \mathfrak{f}_0) = \alpha$.

Then $\mathfrak{f}_0 = N_0 \supseteq N_1 \supseteq \cdots \supseteq N_\omega$.

Each $P_k \in \bigcap \{ \mathfrak{f}_k \mid \mathfrak{f}_k \subseteq \mathfrak{f}_k \}$. Generally iterate each $P_k$ $M_{k+1}$ to absorb $M_{k+1}$ and simultaneously copy the iteration of $M_{k+1}$ into the one of $M_{k+1}$. 
\text{Hom } j_n : \mathbb{N}_n \to \mathbb{N}_{n+1}

L ((\mathbb{R}_n^r, \text{Hom}^{\mathbb{R}_n^r})) = L ((\mathbb{R}_n^r, \text{Hom}^{\mathbb{R}_n^r})) \text{ for } h; \text{ which is}

\text{Cell (} \omega, \alpha \text{)} = \frac{\text{Hom}^{\mathbb{R}_n^r}}{\mathbb{N}_n^r}.

\forall \alpha \geq h \quad j_n (\alpha) = \alpha_0.

A_{\alpha} = \{ \mathbf{e} \in \text{Hom} (\mathbb{R}_n^r, \mathbb{R}_n^r) \mid \text{Cell (} \omega, \alpha \text{)} \}

\prod_{\text{P}(\mathbb{R}_n^r)} A_{\alpha}^{A_{\alpha}} = A