2016-07-26 14:30 NAM TRANG

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JOINT WITH JOHN STEEL

Goal: HOD computation (with lpm) in models of $AD^+ + V = L(\mathcal{P}(\mathbb{R})) + \neg AD_{\mathbb{R}}$.

Caveat (applications)

- $HOD \models GCH + (\forall\kappa)\square_\kappa$
- HOD as a limit of a direct limit system $\mathcal{F}$ s.t. letting $M_\infty = \text{dir lim } \mathcal{F}$, we have: $M_\infty | \Theta = HOD | \Theta$

Comparison of the least branch hierarchy vs rigidly layered hierarchy

LBH

- uniform hierarchy (comparison theory worked out) can prove: $HOD \models GCH, \square, \ldots$
- Core model induction not worked out at all (Easy one: prove $AD^{L(\mathbb{R})}$ using lpm)

RLH

- relies on patterns of in "minimal model theory" (comparisons up to minimal model of LSA. Can prove $GCH + \forall\kappa\, \square_{\kappa,2}$ in HOD.
- Thm (CMI & Sargsyan–T.) $Con(PFA) \to Con(LSA)$.

HOD in $AD_{\mathbb{R}}$-case (under HPC, see below): Suppose $\theta_{\alpha+1} < \Theta$ and (P, Σ) is an lbr hod pair s.t. $(P, \Sigma) \notin \mathbb{B}_{\alpha+1}(\mathbb{R})$. Then

$$HOD | \theta_{\alpha+1} \trianglelefteq M_\infty(P, \Sigma)$$

Furthermore, if (P, Σ) is minimal such then

Def (1) HPC (Hod pair capturing) if A is Suslin co-Suslin then there is an lbr HOD pair (Q, Λ) such that $A \leq_w \mathrm{Code}(\Lambda)$.

(2) Suppose P is an lpm. Let
$$\delta^P = \sup\{ ~~\ldots~~ \, lh(E)+1 \mid E \text{ an extender on the } P\text{-sequence}\}$$
Given $\kappa \in P$:
$$o^P(\kappa) = \sup\{ lh(E)+1 \mid E \text{ on } P\text{-sequence} + crit(E) = \kappa\}$$
P has a top block iff $(\exists \kappa < \delta^P)(o(\kappa) = \delta^P)$

In this case we say that κ begins the top block of P ~~and denote~~ iff $o^P(\kappa) = \delta^P$ and κ is the least such; we write κ^P for this κ.

(Fairly)
Local HOD computation: Suppose $\underset{\sim}{\Gamma}$ is inductive-like and non-selfdual and

$\underset{\sim}{\Delta}_\Gamma \models \mathrm{HPC} + \forall A$ A is Suslin co-Suslin.

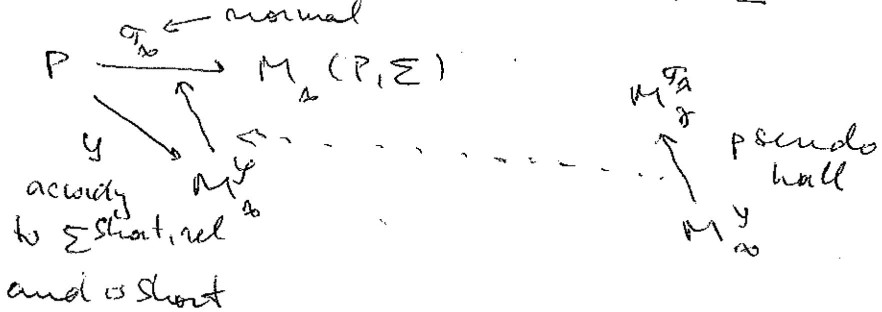
and $\exists (P, \Sigma)$ s.t. $(P, \Sigma) \notin \underset{\sim}{\Delta}_\Gamma$. Then, letting (P, Σ) be $\leq^*$-minimal such and $i : P \to M_\infty(P, \Sigma)$ be the direct limit map then

- P has a top block $[\kappa^P, \delta^P)$
- $i(\kappa^P) = \delta_{\underset{\sim}{\Gamma}}$

[• $i(\delta^P) \geq \Theta_{\underset{\sim}{\Gamma}}$ = the Wadge rank of $\underset{\sim}{\Gamma}$]

- $i(\delta^P) \geq$ the next Suslin cardinal $\gamma > \delta_{\underset{\sim}{\Gamma}}$
- The Wadge rank of Σ is projective in A for any A with Wadge rank γ.

- <u>Why P has a topblock</u>: If not, it is a limit of topblocks each of which starts with a cutpoint. Then $o(M_\infty(P,\Sigma)) \leq \delta_{\tilde\Gamma}$. Then, using this morning's calculation one gets Σ is δ_Γ-Suslin so $\Sigma \in \tilde\Gamma$. (Also follows from the fact that $\delta_{\tilde\Gamma}$ is a regular cardinal.)
- <u>Why is $i(\kappa^P) = \delta^P$</u>: If $<$ then $\Sigma^{\text{short, rel}} \in \tilde\Delta_\Gamma$, because:



Then we can compute Σ^{rel}, and even Σ into $\tilde\Gamma$:

Let $\tilde\Gamma_0 \subseteq \tilde\Delta_\Gamma \subseteq \tilde\Gamma_1$ be inductive-like + scaled s.t. $\Sigma^{\text{short, rel}} \in \tilde\Delta_{\Gamma_0}$. Let $(N^*, \delta_{N^*}, \Sigma_{N^*})$ be Γ_0-Woodin mouse that captures $\Sigma^{\text{sh, rel}}$.
For simplicity assume Γ is a limit of inductive-like pointclasses. Then can show $\exists \langle \gamma, k\rangle < (\delta_{N^*}, 0)$ s.t.
$P \xrightarrow[\sigma]{} (N^{\mathbb{C}}_{(\gamma,k)})^{N^*}$

Let $\tilde\Gamma_0 \subseteq \tilde\Delta_\Gamma \subseteq \tilde\Gamma_1$ be inductive-like scaled pointclasses s.t.

(1) $\Sigma^{\text{short, rel}} \in \tilde\Delta_{\Gamma_0}$

(2) $\Sigma \in \tilde\Delta_{\Gamma_1}$

Let $(N^*, \delta_{N^*}, \Sigma_{N^*})$ be a Γ_1-Woodin mouse that captures $\Sigma^{\text{short,rel}}$, Σ and universal Γ_0, Γ_1-sets.

Let $((\mathcal{N}_{(\gamma,k)}, \Lambda_{\gamma,k}) \mid (\gamma,k) \leq (\delta, 0))$ be the HMC of N^* up to δ where δ is the first Γ_0-Woodin in N^*.

Case 1 $\exists (\gamma,k) < (\delta,0)$ s.t. $\exists i : P \xrightarrow{\mathcal{T}, \Sigma} \mathcal{N}_{(\gamma,k)}$ and $\Sigma_{\mathcal{T}} = \Lambda_{(\gamma,k)}$. Then $\Sigma_{\mathcal{T}}$ must be projective in Γ_0 as the Q-structures are given by C_{Γ_0}. ☒ (Case 1)

Case 2 $\exists i \; P \xrightarrow{\mathcal{T}, \Sigma} M_\infty^{\mathcal{T}} \trianglerighteq \mathcal{N}_{(\delta,0)}$. So $\mathcal{T}$ is short so it is according to $\Sigma^{\text{short,relevant}}$ so we can find Q-structure for δ in $C_{\Gamma_0}(N^* | \delta)$. ☒ Case 2