Math 2E Lec A Solution
Quiz 1

(4 pts) **1)** Find a parametric equation of the line through the points $(1,-1,2)$ and $(-1,0,1)$.

The standard parameterization for the line connecting these two points is
$\mathbf{l}(t) = t\langle -1,0,1\rangle + (1-t)\langle 1,-1,2\rangle$
$\mathbf{l}(0) = \langle 1,-1,2\rangle$ and $\mathbf{l}(1) = \langle -1,0,1\rangle$, so the curve clearly goes through both points.
Rewriting the equation we find $\mathbf{l}(t) = \langle 1,-1,2\rangle + \langle -2,1,-1\rangle t$ which we recognize as a parameterization for a line, so we are done.

As a note, this approach for making a parameterization for a line is very commonly used, so if your approach was more complicated it would be useful to understand this approach.

(6 pts) **2)** Find a parametric representation of the surface $z = x + 2y$.

Two approaches:
The easy algebraic answer is to note that we can rewrite this equation as a function of x and y as $f(x,y) = x + 2y$, which can be easily parameterized as
$\mathbf{r}(s,t) = \langle s,t,f(s,t)\rangle = \langle s,t,s+2t\rangle$

You can also approach this using a more geometric approach: A plane can be thought of as all linear combinations of any two non-parallel vectors that lie on the plane, starting from some point on the plane. We'll find two non-parallel vectors on the plane by finding three points on the plane.
First note that $(0,0,0)$ is on the plane, which simplifies the rest of the calculations. I'll arbitrarily select the points $(-2,1,0)$ and $(-1,1,1)$ which also lie on the plane. Thus, two vectors that lie on the plane $\langle -2,1,0\rangle - \langle 0,0,0\rangle = \langle -2,1,0\rangle$ and $\langle -1,1,1\rangle - \langle 0,0,0\rangle = \langle -1,1,1\rangle$. These vectors are not parallel, so we can use these vectors to help describe the plane.

The plane is also described as $\mathbf{r}(s,t) = s\langle -2,1,0\rangle + t\langle -1,1,1\rangle + \langle 0,0,0\rangle$ or
$\mathbf{r}(s,t) = \langle -2s-t, s+t, t\rangle$.