(4 pts) 1) Find a parametric equation of the line through the points \((1, -1, 2)\) and \((-1, 0, 1)\).

The standard parameterization for the line connecting these two points is

\[
\mathbf{r}(t) = t\langle -1, 0, 1 \rangle + (1 - t)\langle 1, -1, 2 \rangle
\]

\(\mathbf{r}(0) = \langle 1, -1, 2 \rangle\) and \(\mathbf{r}(1) = \langle -1, 0, 1 \rangle\), so the curve clearly goes through both points.

Rewriting the equation we find \(\mathbf{r}(t) = \langle 1, -1, 2 \rangle + \langle -2, 1, -1 \rangle t\) which we recognize as a parameterization for a line, so we are done.

As a note, this approach for making a parameterization for a line is very commonly used, so if your approach was more complicated it would be useful to understand this approach.

(6 pts) 2) Find a parametric representation of the surface \(z = x + 2y\).

Two approaches:

The easy algebraic answer is to note that we can rewrite this equation as a function of \(x\) and \(y\) as \(f(x, y) = x + 2y\), which can be easily parameterized as

\[
\mathbf{r}(s, t) = \langle s, t, f(s, t) \rangle = \langle s, t, s + 2t \rangle
\]

You can also approach this using a more geometric approach: A plane can be thought of as all linear combinations of any two non-parallel vectors that lie on the plane, starting from some point on the plane. We'll find two non-parallel vectors on the plane by finding three points on the plane.

First note that \(\langle 0, 0, 0 \rangle\) is on the plane, which simplifies the rest of the calculations. I'll arbitrarily select the points \((-2, 1, 0)\) and \((-1, 1, 1)\) which also lie on the plane. Thus, two vectors that lie on the plane \(\langle -2, 1, 0 \rangle - \langle 0, 0, 0 \rangle = \langle -2, 1, 0 \rangle\) and \(\langle -1, 1, 1 \rangle - \langle 0, 0, 0 \rangle = \langle -1, 1, 1 \rangle\). These vectors are not parallel, so we can use these vectors to help describe the plane.

The plane is also described as \(\mathbf{r}(s, t) = s\langle -2, 1, 0 \rangle + t\langle -1, 1, 1 \rangle + \langle 0, 0, 0 \rangle\) or \(\mathbf{r}(s, t) = \langle -2s - t, s + t, t \rangle\).