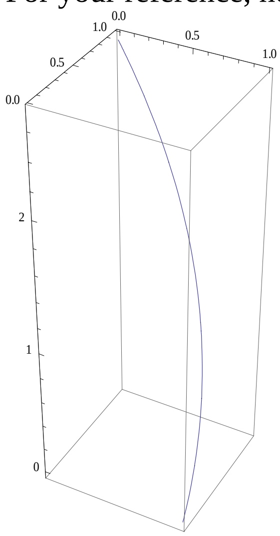
Math 2E Lec A Solution
Quiz 2

1) *(4 pts)* Find the arc-length for the helix $\mathbf{r}(t) = \langle \cos t, \sin t, t\sqrt{3} \rangle$ between the points $(1,0,0)$ and $\left(0, 1, \frac{\pi\sqrt{3}}{2}\right)$.

For your reference, here's a graph of the curve:



By looking at the *z* component of the points, we see that these points correspond to $t = 0$ and $t = \frac{\pi}{2}$

$$s = \int_a^b \left\| \mathbf{r}'(t) \right\| dt$$

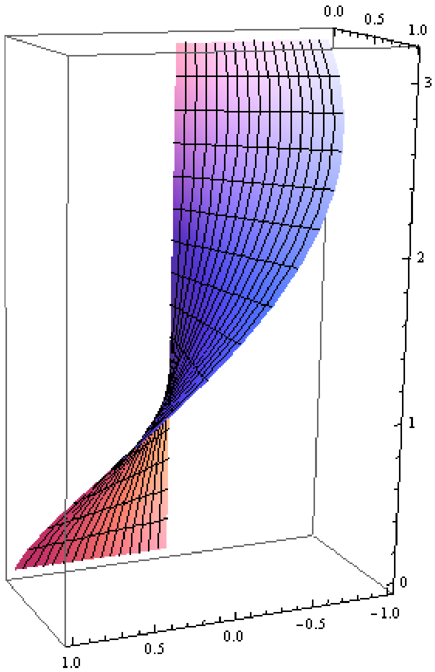
$$\mathbf{r}'(t) = \langle -\sin t, \cos t, \sqrt{3} \rangle$$

$$\left\| \mathbf{r}'(t) \right\| = \sqrt{\sin^2 t + \cos^2 t + 3} = \sqrt{4} = 2$$

$$s = \int_a^b \left\| \mathbf{r}'(t) \right\| dt = \int_0^{\frac{\pi}{2}} 2\, dt = 2t \Big|_0^{\frac{\pi}{2}} = \pi$$

2) *(6 pts)* Find the surface area of the helicoid (spiral ramp) with the vector equation $\mathbf{r}(u,v) = \langle u\cos v, u\sin v, v\rangle$ with $0 \le u \le 1$ and $0 \le v \le \pi$. You may leave your final answer in terms of a <u>single</u> integral (but not a double integral!)

For your reference, here is a graph of the surface:



$$A = \iint_R \|\mathbf{r}_u \times \mathbf{r}_v\| \, dA$$

$\mathbf{r}_u = \langle \cos v, \sin v, 0\rangle$ $\qquad\qquad$ $\mathbf{r}_v = \langle -u\sin v, u\cos v, 1\rangle$

$$\mathbf{r}_u \times \mathbf{r}_v = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \cos v & \sin v & 0 \\ -u\sin v & u\cos v & 1 \end{vmatrix} = \sin v\,\mathbf{i} - \cos v\,\mathbf{j} + \left(u\cos^2 v + u\sin^2 v\right)\mathbf{k} = \langle \sin v, -\cos v, u\rangle$$

$$\|\mathbf{r}_u \times \mathbf{r}_v\| = \sqrt{\sin^2 v + \cos^2 v + u^2} = \sqrt{1+u^2}$$

$$A = \iint_R \|\mathbf{r}_u \times \mathbf{r}_v\| \, dA = \int_0^1 \int_0^\pi \sqrt{1+u^2}\, dv\, du = \pi \int_0^1 \sqrt{1+u^2}\, du$$

This is an integral that you might be able to do (it requires a trig substitution and some work), but I didn't want you to have to take the time to do so. Numerically, the answer is ≈ 3.6.