1) (4 pts) Find the arc-length for the helix \( \mathbf{r}(t) = (\cos t, \sin t, t\sqrt{3}) \) between the points \((1, 0, 0)\) and \(\left(0, 1, \frac{\pi \sqrt{3}}{2}\right)\).

For your reference, here's a graph of the curve:

By looking at the \(z\) component of the points, we see that these points correspond to \(t = 0\) and \(t = \frac{\pi}{2}\).

\[
s = \int_a^b \| \mathbf{r}'(t) \| \, dt
\]

\[
\mathbf{r}'(t) = (-\sin t, \cos t, \sqrt{3})
\]

\[
\| \mathbf{r}'(t) \| = \sqrt{\sin^2 t + \cos^2 t + 3} = \sqrt{4} = 2
\]

\[
s = \int_0^{\frac{\pi}{2}} 2 \, dt = 2t \Bigg|_0^{\frac{\pi}{2}} = \pi
\]
2) (6 pts) Find the surface area of the helicoid (spiral ramp) with the vector equation \( \mathbf{r}(u, v) = \langle u \cos v, u \sin v, v \rangle \) with \( 0 \leq u \leq 1 \) and \( 0 \leq v \leq \pi \). You may leave your final answer in terms of a single integral (but not a double integral!)

For your reference, here is a graph of the surface:

\[
A = \iint_R \| \mathbf{r}_u \times \mathbf{r}_v \| \, dA \\
\mathbf{r}_u = \langle \cos v, \sin v, 0 \rangle \quad \quad \quad \mathbf{r}_v = \langle -u \sin v, u \cos v, 1 \rangle \\
\mathbf{r}_u \times \mathbf{r}_v = \begin{vmatrix}
\mathbf{i} & \mathbf{j} & \mathbf{k} \\
\cos v & \sin v & 0 \\
u \sin v & u \cos v & 1
\end{vmatrix} = \sin v \mathbf{i} - \cos v \mathbf{j} + u \cos^2 v + u \sin^2 v \mathbf{k} = \langle \sin v, -\cos v, u \rangle \\
\| \mathbf{r}_u \times \mathbf{r}_v \| = \sqrt{\sin^2 v + \cos^2 v + u^2} = \sqrt{1 + u^2} \\
A = \iint_R \| \mathbf{r}_u \times \mathbf{r}_v \| \, dA = \int_0^1 \int_0^\pi \sqrt{1 + u^2} \, dv \, du = \pi \int_0^1 \sqrt{1 + u^2} \, du
\]

This is an integral that you might be able to do (it requires a trig substitution and some work), but I didn't want you to have to take the time to do so. Numerically, the answer is \( \approx 3.6 \).