1) (5 pts) Determine whether or not the vector field is conservative. If it is, find the potential function.
\[ \mathbf{F}(x, y) = \langle y \cos x, \sin x - y \rangle \]

We could start by noticing that \( \mathbf{F}(x, y) = \langle y \cos x, \sin x - y \rangle = \langle M(x, y), N(x, y) \rangle \) could be conservative, because \( M_y = \cos x = N_x = \cos x \).

Next, if there were a potential function, \( f(x, y) \), then \( \nabla f = \mathbf{F} \). In other words, \( f_x(x, y) = y \cos x \) and \( f_y(x, y) = \sin x - y \). Solving for the function, \( f(x, y) = \int M \, dx = \int y \cos x \, dx = y \sin x + G(y) \). We also know that the \( y \)-partial of this function has to be \( N(x, y) \), so \( f_y(x, y) = \sin x + G'(y) = N(x, y) = \sin x - y \). Solving for the unknown function, \( G'(y) = -y \). \( G(y) = \int G'(y) \, dy = \int -y \, dy = \frac{1}{2} y^2 + c \) (where \( c \) is some unknown constant).

Thus the potential function is \( f(x, y) = y \sin x + \frac{1}{2} y^2 + c \)

We can verify this answer by noting that \( \nabla f = \mathbf{F} \). Because there is a potential function for this vector field, the vector field is conservative.

2) (5 pts) Evaluate the line integral:
\[ \int_C (2x - y) \, ds \]

Where \( C \) is the quarter circle from \((2,0)\) to \((0,2)\) centered at the origin.

A parameterization for this path is \( \mathbf{γ}(t) = \langle 2 \cos t, 2 \sin t \rangle \) where \( 0 \leq t \leq \frac{\pi}{2} \).
\[ \mathbf{γ}'(t) = \langle -2 \sin t, 2 \cos t \rangle \] so \( \| \mathbf{γ}'(t) \| = \sqrt{4 \sin^2 t + 4 \cos^2 t} = \sqrt{4 (\sin^2 t + \cos^2 t)} = 2 \).

We are integrating the function \( f(x, y) = 2x - y \), so
\[
\int_C f \, ds = \int_0^{\pi/2} f(\mathbf{γ}(t)) \| \mathbf{γ}'(t) \| \, dt = \int_0^{\pi/2} \left( 2(2 \cos t) - (2 \sin t) \right) \frac{2}{\sqrt{4}} \, dt = 4 \int_0^{\pi/2} (2 \cos t - \sin t) \, dt
\]
\[ = 4 \left[ 2 \sin t + \cos t \right]_0^{\pi/2} = 4 (2 - 1) = 4 \]