Math 2E Lec A Solution
Quiz 3

1) *(5 pts)* Determine whether or not the vector field is conservative. If it is, find the potential function.
$\mathbf{F}(x,y) = \langle y\cos x, \sin x - y\rangle$

We could start by noticing that $\mathbf{F}(x,y) = \langle y\cos x, \sin x - y\rangle = \langle M(x,y), N(x,y)\rangle$ could be conservative, because $M_y = \cos x = N_x = \cos x$.

Next, if there were a potential function, $f(x,y)$, then $\nabla f = \mathbf{F}$. In other words, $f_x(x,y) = y\cos x$ and $f_y(x,y) = \sin x - y$. Solving for the function, $f(x,y) = \int M\,dx = \int y\cos x\,dx = y\sin x + G(y)$. We also know that the *y*-partial of this function has to be $N(x,y)$, so $f_y(x,y) = \sin x + G'(y) = N(x,y) = \sin x - y$. Solving for the unknown function, $G'(y) = -y$. $G(y) = \int G'(y)\,dy = \int -y\,dy = \frac{1}{2}y^2 + c$ (where c is some unknown constant).

Thus the potential function is $f(x,y) = y\sin x + \frac{1}{2}y^2 + c$
We can verify this answer by noting that $\nabla f = \mathbf{F}$. Because there is a potential function for this vector field, the vector field is conservative.

2) *(5 pts)* Evaluate the line integral:
$\int_C (2x - y)\,ds$
Where C is the quarter circle from $(2,0)$ to $(0,2)$ centered at the origin.

A parameterization for this path is $\boldsymbol{\gamma}(t) = \langle 2\cos t, 2\sin t\rangle$ where $0 \le t \le \frac{\pi}{2}$.
$\boldsymbol{\gamma}'(t) = \langle -2\sin t, 2\cos t\rangle$ so $\|\boldsymbol{\gamma}'(t)\| = \sqrt{4\sin^2 t + 4\cos^2 t} = \sqrt{4\left(\sin^2 t + \cos^2 t\right)} = 2$.

We are integrating the function $f(x,y) = 2x - y$, so

$$\int_C f\,ds = \int_0^{\frac{\pi}{2}} f(\boldsymbol{\gamma}(t))\underbrace{\|\boldsymbol{\gamma}'(t)\|\,dt}_{ds} = \int_0^{\frac{\pi}{2}} \left(2\underbrace{(2\cos t)}_{x} - \underbrace{(2\sin t)}_{y}\right)\underbrace{2}_{\|\gamma'(t)\|}\,dt = 4\int_0^{\frac{\pi}{2}} (2\cos t - \sin t)\,dt$$
$$= 4\left[2\sin t + \cos t\right]_0^{\pi/2} = 4(2-1) = 4$$