Math 2E Lec A Solution
Quiz 5

1) *(5 pts)* Let $\mathbf{F}(x,y,z)=\langle L(x,y,z), M(x,y,z), N(x,y,z)\rangle$, be a vector field whose components $L(x,y,z)$, $M(x,y,z)$ and $N(x,y,z)$ have continuous first and second order partial derivatives (that is, $\mathbf{F}$ is C^2). Show $\operatorname{div}(\operatorname{curl}\mathbf{F})=0$.

First we calculate the curl:

$$\operatorname{curl}\mathbf{F}=\nabla\times\mathbf{F}=\begin{vmatrix}\mathbf{i} & \mathbf{j} & \mathbf{k}\\ \partial/\partial x & \partial/\partial y & \partial/\partial z\\ L & M & N\end{vmatrix}=\langle N_y-M_z, L_z-N_x, M_x-L_y\rangle \quad \textit{(2 pts)}$$

Now the divergence of the resulting vector field:

$$\operatorname{div}(\operatorname{curl}\mathbf{F})=\nabla\cdot(\operatorname{curl}\mathbf{F})=\nabla\cdot\langle N_y-M_z, L_z-N_x, M_x-L_y\rangle=(N_{yx}-M_{zx})+(L_{zy}-N_{xy})+(M_{xz}-L_{yz})$$
$$=(N_{yx}-N_{xy})+(L_{zy}-L_{yz})+(M_{xz}-M_{zx})$$

(1 pt)

Now, because L, M and N have continuous first and second order partial derivatives $N_{yx}=N_{xy}, L_{zy}=L_{yz}, M_{xz}=M_{zx}$ leaving us with $\operatorname{div}(\operatorname{curl}\mathbf{F})=0$ *(2 pts)*

2) *(5 pts)* Find the divergence and the curl of the vector field $\mathbf{F}(x,y,z)=\langle y^2z, e^{xyz}, x^2y\rangle$.

$$\operatorname{div}(\mathbf{F})=\nabla\cdot\mathbf{F}=\nabla\cdot\langle y^2z, e^{xyz}, x^2y\rangle=0+xze^{xyz}+0=xze^{xyz} \quad \textit{(2 pts)}$$

$$\operatorname{curl}\mathbf{F}=\nabla\times\mathbf{F}=\begin{vmatrix}\mathbf{i} & \mathbf{j} & \mathbf{k}\\ \partial/\partial x & \partial/\partial y & \partial/\partial z\\ y^2z & e^{xyz} & x^2y\end{vmatrix}=\langle x^2-xye^{xyz}, y^2-2xy, yze^{xyz}-2yz\rangle \quad \textit{(3 pts)}$$