Sample solution: Book, Exercises for Section 15.6, problem 50.

I give only main points, please fill in the details.

The surface S is the portion of the plane $x + 2y + z = 4$ above the region in the xy-plane bounded by $y = x^2$ and $y = 1$; denote this reagion by A. The density is given by $\rho(x, y, z) = y$.

Draw the picture. The region A in the xy-plane has the following bounds: $-1 \leq x \leq 1$ and $x^2 \leq y \leq 1$. The surface is given by function $f(x, y) = z$, i.e. $f(x, y) = 4 - x - 2y$ so the evaluation formula (2) from the "brief guide" can easily be used. (And seems to be more appropiate for this problem.)

We have $f_x(x, y, z) = -1$ and $f_y(x, y, z) = -2$ so $1 + f_x^2 + f_y^2 = 6$. Hence the mass M is given by

$$M = \int\int_A \rho(x, y, z)\sqrt{1 + f_x^2 + f_y^2}\ \mathrm{d}x\mathrm{d}y = \int\int_A y\sqrt{6}\ \mathrm{d}x\mathrm{d}y = \sqrt{6}\int_{-1}^{1}\int_{x^2}^{1} y\ \mathrm{d}y\mathrm{d}x$$

The last integral is easy to evaluate.

Next we compute the center of mass $(\bar{x}, \bar{y}, \bar{z})$ of S. Recall that $\bar{x}$ is given by M_x/M where

$$M_x = \int\int_A x\rho(x, y, z)\sqrt{1 + f_x^2 + f_y^2}\ \mathrm{d}x\mathrm{d}y$$

and $\bar{y}, \bar{z}$ are computed similarly. I will only compute $\bar{x}$ and leave the computation of $\bar{y}, \bar{z}$ to you as exercise. By substituting into the formula above:

$$M_x = \int\int_A xy\sqrt{6}\ \mathrm{d}x\mathrm{d}y = \sqrt{6}\int_{-1}^{1}\int_{x^2}^{1} xy\ \mathrm{d}y\mathrm{d}x$$

and the last integral is easy to evaluate.