MATH 2E WINTER 2018 HOMEWORK 1

Due: Thursday, January 25, 2018 Please turn in in the discussion.

If you work in a group of two please turn in only one paper, but put both names and Student ID's on the paper. If you work alone, turn in a paper with your name and Student ID only.

Student name/id (include all students in the group):

IMPORTANT INSTRUCTIONS: It is crucial that you write your solutions clearly and that each solution clearly shows how you arrive at the results. This is the point of homeworks – to practice understanding of the material, clear writing, and the ability to express your understanding. Also, try to write your solutions as efficiently as possible, meaning that you should judge what to write and what not. Writing irrelevant text makes the solution confused and difficult to understand and grade. Leaving out important points makes the solution incomplete. So you need to judge what is relevant and what not. Often there is no recipe for this; the ability to recognize what to write and what not is a skill which needs to be developed, and this course (= Math 2E) is intended to help you with this.

The single integrals in this assignment are quite easy to evaluate, so please try to evaluate to the end. Do not use calculators. Instead, leave the results in the symbolic form, for instance if the result is of the form $\pi(1-3\sqrt{2})$, leave it in this form.

1. (10pt) Use polar coordinates to calculate the area of one lobe of the lamina in the plane given by equation $r = \cos 4\theta$. Draw the picture.

2. (10pt) Evaluate the triple integral

$$\int\int\int_E xy \, \mathsf{d}V$$

where E is the region in the 3-dimensional space:

(a) between the planes $y = 1$, $y = \sqrt{5}$,
(b) above the plane YZ, so $x \geq 0$, and
(c) inside the sphere with center $(0, 0, 0)$ and radius 3.

Use the direct method where you transform this triple integral into three single integrals, interpreting E as a region of the right type. Draw the picture!

3. (10pt) The region E is the upper half of the ball centered in $(0,0,0)$ with radius a. Assume the density function is constant, say $\sigma(x,y,z) = K$.

(a) Use cylindrical coordinates to calculate the mass of E.
(b) Use cylindrical coordinates to calculate the moment M_{xy} of E. Then calculate $\bar{z}$.
(c) Determine the center of gravity of E. Notice that in order to determine the coordinates $\bar{x}$ and $\bar{y}$ you do not need to do the hard work to evaluate integrals for M_{xz} and M_{yz}, but the value of the integrals can be determined without calculation by looking at the shape of region E and the density function. Determine these values without calculations and give explanation to your solution.

4. (10pt) The region E is the same as in Problem 3, that is, the upper half of the ball centered in $(0,0,0)$ with radius a. This time the density function is not constant, but is given by formula

$$\sigma(x,y,z) = K\sqrt{x^2+y^2+z^2}$$

where K is a constant.

(a) Use spherical coordinates to calculate the mass of E.
(b) Use spherical coordinates to calculate the moment M_{xy} of E. Then calculate $\bar{z}$.
(c) Determine the center of gravity of E. Can you use the same reasoning when determining M_{xz} and M_{yz} as in (c) in Problem 3? Explain why yes or why not, depending on your answer.