MATH 2E WINTER 2018 HOMEWORK 2

Due: Thursday, February 1, 2018 Please turn in in the discussion.

If you work in a group of two please turn in only one paper, but put both names and Student ID's on the paper. If you work alone, turn in a paper with your name and Student ID only.

Student name/id (include all students in the group):

IMPORTANT INSTRUCTIONS: It is crucial that you write your solutions clearly and that each solution clearly shows how you arrive at the results. This is the point of homeworks – to practice understanding of the material, clear writing, and the ability to express your understanding. Also, try to write your solutions as efficiently as possible, meaning that you should judge what to write and what not. Writing irrelevant text makes the solution confused and difficult to understand and grade. Leaving out important points makes the solution incomplete. So you need to judge what is relevant and what not. Often there is no recipe for this; the ability to recognize what to write and what not is a skill which needs to be developed, and this course (= Math 2E) is intended to help you with this.

The single integrals in this assignment are quite easy to evaluate, so please try to evaluate to the end. Do not use calculators. Instead, leave the results in the symbolic form, for instance if the result is of the form $\pi(1 - 3\sqrt{2})$, leave it in this form.

1. (10pt) The region E in the 3-dimensional space $\mathbb{R}^3$ is surrounded by the surface with equation

$$x^2 + y^2 = z^4, \tag{1}$$

the xy-plane, and the plane $z = a$ where $a > 0$ is a constant.

(a) Draw a sketch of the surface in (1).
(b) Express the equation of the surface in (1) in cylindrical coordinates. Also give the bounds for cylindrical coordinates.
(c) Calculate the volume of E.
(d) Assume the density function is constant, i.e. $\sigma(x, y, z) = K$ where $K > 0$. Calculate the mass of E.
(e) For the density function as in (d), calculate the moments and the center of gravity of E. Is it possible to determine any of the moments without performing a calculation?

2. (10pt) Evaluate the double integral

$$\int\int_R \frac{x+3y}{(3x+y)(3x+y-1)}\,\mathsf{d}x\mathsf{d}y$$

where R is the inside of the triangle in the plane $\mathbb{R}^2$ given by corner points

$$(3/8,-1/8),\ (3/4,-1/4),\ (1/2,1/2).$$

Use a suitable linear change of coordinates. Sketch the situation.

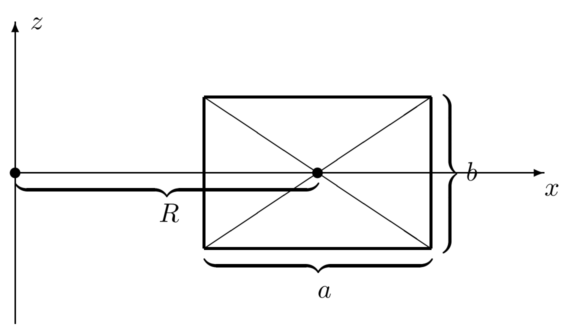
3. (10pt) Consider the object in the form of a region in the 3-dimensional space $\mathbb{R}^3$ above the xy-plane and below the ellipsoid with equation

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$$

where $a,b,c>0$, with constant density $\sigma(x,y,z)=K$. Calculate the mass and the center of gravity of E. When calculating the center of gravity, look at which of the coordinates $\bar{x},\bar{y},\bar{z}$ really needs a calculation and which can be deduced based on the properties of E and σ.

For your calculations use the change of coordinates which is a suitable minor modification of spherical coordinates. Find out what this change of coordinates is; this is an important part of the solution.

4. (10pt) Consider the surface generated in the 3-dimensional space $\mathbb{R}^3$ by revolving a rectangle with sides of lengths a and b about axis z. The side of length a is parallel to axis y, and the side of length b is parallel to axis z. The distance of the center of the rectangle from the origin $(0,0,0)$ is R; here the center is the point where the two diagonals intersect. See the picture below.



So the situation resembles to that in the case of a torus which we discussed in the lecture, but this time it is a revolution of a rectangle instead of a revolution of a circle.

Calculate the volume of the region E in $\mathbb{R}^3$ surrounded by this surface (that is, the inside of the surface). Here use a change of coordinates analogous to that we used in the case of torus in the lecture. **Warning:** the change of coordinates is similar, but not the same. You need to figure out what the difference is.