MATH 2E WINTER 2018 HOMEWORK 4

Due: Thursday, March 1, 2018 Please turn in in the discussion.

If you work in a group of two please turn in only one paper, but put both names and Student ID's on the paper. If you work alone, turn in a paper with your name and Student ID only.

Student name/id (include all students in the group):

IMPORTANT INSTRUCTIONS: It is crucial that you write your solutions clearly and that each solution clearly shows how you arrive at the results. This is the point of homeworks – to practice understanding of the material, clear writing, and the ability to express your understanding. Also, try to write your solutions as efficiently as possible, meaning that you should judge what to write and what not. Writing irrelevant text makes the solution confused and difficult to understand and grade. Leaving out important points makes the solution incomplete. So you need to judge what is relevant and what not. Often there is no recipe for this; the ability to recognize what to write and what not is a skill which needs to be developed, and this course (= Math 2E) is intended to help you with this.

The single integrals in this assignment are quite easy to evaluate, so please try to evaluate to the end. Do not use calculators. Instead, leave the results in the symbolic form, for instance if the result is of the form $\pi(1-3\sqrt{2})$, leave it in this form.

1. (5pt) Given is a vector field $\mathbf{F}(x,y,z)$ in $\mathbb{R}^3$ defined by

$$\mathbf{F}(x,y,z) = yze^{xz}\cdot\mathbf{i} + (z+e^{xz})\cdot\mathbf{j} + (1+y+xye^{xz})\cdot\mathbf{k}$$

Also given is a spiral C in $\mathbb{R}^3$ parametrized in cylindrical coordinates as follows:

$$r(\theta) = e^{\theta}-1 \quad z(\theta)=\theta \qquad 0\le\theta\le 2\pi.$$

(a) Determine whether $\mathbf{F}$ is conservative, and if so, calculate the potential function.

(b) Interpreting $\mathbf{F}$ as a force field, find the work done by $\mathbf{F}$ when $\mathbf{F}$ moves an object from the beginning of spiral C to its end.

2. (5pt) Given is a force field $\mathbf{F}(x,y)$ in the plane defined by

$$\mathbf{F}(x,y) = \left(\frac{y^2}{2\sqrt{1+x^2}} - y^3\right)\cdot\mathbf{i} + y\ln(x+\sqrt{1+x^2})\cdot\mathbf{j}$$

Consider the situation that $\mathbf{F}$ pushes an object along the circle centered in the origin with radius a in the **clockwise direction**. Use Green's theorem to calculate work done by the field $\mathbf{F}$ after one loop.

3. (5pt) Use Green's theorem to calculate the area of the region surrounded by the closed curve C in the plane with equation

$$x^{2/3} + y^{2/3} = 1$$

For this, you will need to find the right parametrization of C.

4. (5pt) Consider the open region D in the plane $\mathbb{R}^2$ which is obtained by removing the point $(0,0)$. In other words, D consists of all points (x,y) in the plane except $(0,0)$. A vector field $\mathbf{F}(x,y) = P(x,y)\cdot\mathbf{i} + Q(x,y)\cdot\mathbf{j}$ is defined on D as follows:

$$\mathbf{F}(x,y) = \frac{x-y}{x^2+y^2}\cdot\mathbf{i} + \frac{x+y}{x^2+y^2}\cdot\mathbf{j}.$$

(a) Show that $\partial P/\partial y = \partial Q/\partial x$.

(b) Consider any piecewise smooth closed simple positively oriented curve C such that the point $(0,0)$ is **not** surrounded by C. Evaluate $\int_C \mathbf{F}d\mathbf{r}$. If you can use Green's theorem, use it, but explain how are you using it and why you can use it.

(c) Now consider any piecewise smooth closed simple positively oriented curve C such that the point $(0,0)$ **is** surrounded by C. Evaluate $\int_C \mathbf{F}d\mathbf{r}$. If you can use Green's theorem, use it, but explain how are you using it and why you can use it.

(d) Is the field $\mathbf{F}(x,y)$ conservative on D? Justify your answer.