MATH 2E WINTER 2018 HOMEWORK

Due: Thursday, March 15, 2018 Please turn in in the discussion.

If you work in a group of two please turn in only one paper, but put both names and Student ID's on the paper. If you work alone, turn in a paper with your name and Student ID only.

Student name/id (include all students in the group):

IMPORTANT INSTRUCTIONS: It is crucial that you write your solutions clearly and that each solution clearly shows how you arrive at the results. This is the point of homeworks – to practice understanding of the material, clear writing, and the ability to express your understanding. Also, try to write your solutions as efficiently as possible, meaning that you should judge what to write and what not. Writing irrelevant text makes the solution confused and difficult to understand and grade. Leaving out important points makes the solution incomplete. So you need to judge what is relevant and what not. Often there is no recipe for this; the ability to recognize what to write and what not is a skill which needs to be developed, and this course (= Math 2E) is intended to help you with this.

The single integrals in this assignment are quite easy to evaluate, so please try to evaluate to the end. Do not use calculators. Instead, leave the results in the symbolic form, for instance if the result is of the form $\pi(1-3\sqrt{2})$, leave it in this form.

1. (5pt) Consider the object S in 3-dimensional space which has the shape of the portion of a helicoid given parametrically by

$$\mathbf{r}(u,v) = u\cos v\cdot\mathbf{i} + u\sin v\cdot\mathbf{j} + v\cdot\mathbf{k}, \qquad 0\le u\le 1 \text{ and } 0\le v\le\pi.$$

The object is made of a material with density function given by

$$\sigma(x,y,z) = z.$$

(a) Sketch the shape of S. Draw examples of grid lines.
(b) Calculate the mass of S.
(c) Calculate the center of gravity of S. Perform only calculations for those coordinates which cannot be easily determined from symmetry properties. For coordinates which can be easily determined from symmetry properties, determine the value of each such coordinate, and provide an explanation why the value is the one you determined. In this case, write down all factors

which determine the value of the coordinate, and explain how. Writing a vague statement of the kind "Because of symmetry" is not sufficient.

Remark.

$$\int \sqrt{1+z^2}\,\mathrm{d}z = \frac{1}{2}\left(\ln(z+\sqrt{1+z^2}) + z\sqrt{1+z^2}\right) + C$$

2. (5pt) Consider the hemisphere with radius a described by

$$x^2 + y^2 + z^2 = a^2 \qquad y \geq 0$$

and a flow of fluid given by vector field

$$\mathbf{F}(x,y,z) = x \cdot \mathbf{i} + y \cdot \mathbf{j} + z \cdot \mathbf{k}.$$

Calculate the rate of flow of fluid passing through the hemisphere from "inside out". Sketch a picture what "inside out" means here. Do not forget about the orientation of the hemisphere!

3. (5pt) Consider the triangle in the 3-dimensional space $\mathbb{R}^3$ with edges

$$(1,0,0), (0,1,0), (0,0,1)$$

and a force field given by

$$\mathbf{F}(x,y,z) = (y^2z + \sqrt{1+x^2}) \cdot \mathbf{i} + (z^2x + \sqrt{1+y^2}) \cdot \mathbf{j} + (x^2y + \sqrt{1+z^2}) \cdot \mathbf{k}$$

The force field $\mathbf{F}$ pushes an object which makes one loop counterclockwise around this triangle. Use Stokes' theorem to calculate the work done by the field $\mathbf{F}$.

4. (5pt) The temperature in the 3-dimensional space $\mathbb{R}^3$ is given by the temperature function

$$T(x,y,z) = x^4 + y^4 + z^4 + xyz$$

Use the Divergence theorem to calculate the rate of heat flow through the sphere with radius a centered in the origin. The conductivity of the environment is constant with value $K > 0$.