MATH 13 WINTER 2017 HOMEWORK 1

Due: Friday January 27, 2017 Please turn in at the lecture.

If you work in a group of two please turn in only one paper, but put both names and Student ID's on the paper. If you work alone, turn in a paper with your name and Student ID only.

Student name/id (include all students in the group):

IMPORTANT INSTRUCTIONS: It is crucial that you write your arguments clearly and that each argument clearly shows how you arrive at the conclusions from the assumptions. This is the point of homeworks – to practice understanding of the material, proofwriting, and the ability to express your understanding. Also, try to write your arguments as efficiently as possible, meaning that you should judge what to write and what not. Writing irrelevant text makes the argument confused and difficult to understand. Leaving out important points makes the argument incomplete. So you need to judge what is relevant and what not. There is no recipe for this; the ability to recognize what to write and what not is a skill which needs to be developed, and this course (= Math 13) is intended to help you with this.

When preparing the homeworks, please follow the **Rules for homeworks** on the course webpage under **Course information and policies** and also the guidelines under **Grading**. In particular, keep in mind the **Aspects of grading** in the **Grading** section.

1. (5pt) Consider the following sets.

$$A = \{0, 1, 2\} \quad B = \{2, 3, 4\} \quad C = \{0, 4, 5\}$$
$$D = \{6\} \quad E = \{1, A, C\}, \quad F = \{B, C, 6\}$$

In each of the following cases express the given set in the roster notation. Do not write any proof, just write the result.

(a) **(1pt)** $A \cap E$
(b) **(1pt)** $A \setminus D$
(c) **(1pt)** $E \setminus F$
(d) **(1pt)** $A \triangle C$
(e) **(1pt)** $A \cup E$

2. (5pt) Write the following sets using the separation method, or equivalently, in builder notation. Do **not** use sets E, O, P and Q used in the discussions and lectures, instead use symbols $<, \leq$, symbols for arithmetic operations and possibly divisibility.

(a) **(1pt)** The set of all positive even integers smaller than 1000.
(b) **(1pt)** The set of all positive divisors of 10000.
(c) **(2pt)** The set of all positive integers which can be expressed as products of two distinct primes.
(d) **(1pt)** The set of all integers which can be expressed as sums of two (not necessarily distinct) squares.

3. (5pt) Prove that for every $a \in \mathbb{Z}$, the fourth power a^4 gives remainder 0 or 1 when divided by 8.

This is a proof by cases. Here follows the analogous proof from the lecture. You should write the proof so that you will consider only **two** cases which cover all options.

Recall that a number b gives remainder 1 when divided by 8 iff there exists some $k \in \mathbb{Z}$ such that $b = 8 \cdot k + 1$. Analogously for remainder 0; obviously the remainder is 0 iff b is divisible by 8.