MATH 13 WINTER 2017 HOMEWORK 2

Due: Friday February 3, 2017

If you work in a group of two please turn in only one paper, but put both names and Student ID's on the paper. If you work alone, turn in a paper with your name and Student ID only.

Student name/id (include all students in the group):

IMPORTANT INSTRUCTIONS: It is crucial that you write your arguments clearly and that each argument clearly shows how you arrive at the conclusions from the assumptions. This is the point of homeworks – to practice understanding of the material, proofwriting, and the ability to express your understanding. Also, try to write your arguments as efficiently as possible, meaning that you should judge what to write and what not. Writing irrelevant text makes the argument confused and difficult to understand. Leaving out important points makes the argument incomplete. So you need to judge what is relevant and what not. There is no recipe for this; the ability to recognize what to write and what not is a skill which needs to be developed, and this course (= Math 13) is intended to help you with this.

When preparing the homeworks, please follow the **Rules for homeworks** on the course webpage under **Course information and policies** and also the guidelines under **Grading**. In particular, keep in mind the **Aspects of grading** in the **Grading** section.

IMPORTANT: You can use any result from the lecture in your solutions. If you are using a result, just quote it (i.e. say the result is from the lecture) – do not prove it.

1. (5pt) Recall that given an integer n, the set $n\mathbb{Z}$ is defined as follows

$$n\mathbb{Z} = \{n \cdot k \mid k \in \mathbb{Z}\} = \{x \in \mathbb{Z} \mid (\exists k \in \mathbb{Z})(x = k \cdot n)\}$$

Prove:

$$m \text{ is a divisor of } n \implies n\mathbb{Z} \subseteq m\mathbb{Z}.$$

Hint. First experiment a bit with concrete numbers: Consider for instance $m = 3$ and $n = 6$ and look what happens.

2. (5pt) Let X, Y be sets and $A, B \subseteq X$. Prove

(a) **(3pt)** $A \smallsetminus B = A \cap B^c$

(b) **(2pt)** If $\mathcal{P}(X) \subseteq \mathcal{P}(Y)$ then $X \subseteq Y$.

Recall that $\mathcal{P}(X)$ is the power set of X.

3. (5pt) Decide if the following statements are true for arbitrary sets A, B. Prove those which are true and disprove those which are false.

(a) **(2pt)** $\mathcal{P}(A \cap B) \subseteq \mathcal{P}(A) \cap \mathcal{P}(B)$.

(b) **(2pt)** $\mathcal{P}(A \cup B) \subseteq \mathcal{P}(A) \cup \mathcal{P}(B)$.

(c) **(1pt)** $\mathcal{P}(A \times B) = \mathcal{P}(A) \times \mathcal{P}(B)$.

Hint. Again experiment first a bit. Write down two sets, for example $A = \{1, 2\}$ and $B = \{3, 4\}$ and try to see what happens. Then approach the general statement.

Recall that to prove that two sets are not equal it suffices to produce an element which is in one of them and not in the other one. Also, to prove that set A is not a subset of B it suffices to find an element of A which is not an element of B.

Finally, to prove that a given statement is not true for arbitrary sets A, B it suffices to find **some** sets A, B for which the statement is false.