MATH 13 WINTER 2017 HOMEWORK 4

Due: Wednesday, March 1, 2017 Please turn in in the lecture.

If you work in a group of two please turn in only one paper, but put both names and Student ID's on the paper. If you work alone, turn in a paper with your name and Student ID only.

Student name/id (include all students in the group):

IMPORTANT INSTRUCTIONS: It is crucial that you write your arguments clearly and that each argument clearly shows how you arrive at the conclusions from the assumptions. This is the point of homeworks – to practice understanding of the material, proofwriting, and the ability to express your understanding. Also, try to write your arguments as efficiently as possible, meaning that you should judge what to write and what not. Writing irrelevant text makes the argument confused and difficult to understand. Leaving out important points makes the argument incomplete. So you need to judge what is relevant and what not. There is no recipe for this; the ability to recognize what to write and what not is a skill which needs to be developed, and this course (= Math 13) is intended to help you with this.

When preparing the homeworks, please follow the **Rules for homeworks** on the course webpage under **Course information and policies** and also the guidelines under **Grading**. In particular, keep in mind the **Aspects of grading** in the **Grading** section.

1. (5pt) Use induction to prove that for every odd $n \in \mathbb{N}$ the sum $11^n + 6^n$ is divisible by 17.

2. (5pt) Use induction to prove that for every $n \in \mathbb{N} \cup \{0\}$, if A is a set with n elements then $\mathcal{P}(A)$ has 2^n elements.

3. (5pt)

(a) **(1pt)** Assume that $f : A \to B$ is an injective function. Prove that there exists a function $g : B \to A$ such that $g \circ f = \mathsf{id}_A$. (Recall that $\mathsf{id}_A : A \to A$ is the identity function defined by $\mathsf{id}_A(a) = a$.)

(b) **(1+1pt)** Let X be a set which has at least two elements. Let $A \subsetneq X$ be a fixed nonempty proper subset of X. Consider a function $h : \mathcal{P}(X) \to \mathcal{P}(X)$ defined by $h(B) = A \cup B$. Is h injective? Is h surjective? In either case prove or disprove.

(c) **(1+1pt)** Let A, B be two disjoint sets and $f : A \to C$ and $g : B \to C$ be functions. Prove that $f \cup g$ is a function, and in fact $f \cup g : A \cup B \to C$. Now assume $A \cap B$ is nonempty. Give examples of functions $f : A \to C$ and $g : B \to C$ such that $f \cup g$ is not a function.