MATH 13 WINTER 2017 HOMEWORK 5

Due: Wednesday March 8, 2017

If you work in a group of two please turn in only one paper, but put both names and Student ID's on the paper. If you work alone, turn in a paper with your name and Student ID only.

Student name/id (include all students in the group):

IMPORTANT INSTRUCTIONS: It is crucial that you write your arguments clearly and that each argument clearly shows how you arrive at the conclusions from the assumptions. This is the point of homeworks – to practice understanding of the material, proofwriting, and the ability to express your understanding. Also, try to write your arguments as efficiently as possible, meaning that you should judge what to write and what not. Writing irrelevant text makes the argument confused and difficult to understand. Leaving out important points makes the argument incomplete. So you need to judge what is relevant and what not. There is no recipe for this; the ability to recognize what to write and what not is a skill which needs to be developed, and this course (= Math 13) is intended to help you with this.

When preparing the homeworks, please follow the **Rules for homeworks** on the course webpage under **Course information and policies** and also the guidelines under **Grading**. In particular, keep in mind the **Aspects of grading** in the **Grading** section.

1. (5pt)

(a) **(2pt)** Use Euclid Algorithm to calculate $\gcd(284, 128)$.

(b) **(3pt)** Calculate $500^{100} \mod 9$.

2. (5pt)

(a) **(2pt)** In the lecture we proved that if $a_1 \equiv a_2 \mod n$ and $b_1 \equiv b_2 \mod n$ then $a_1 \cdot b_1 \equiv a_2 \cdot b_2 \mod n$.

Use this to prove, by induction on k, that if $a_1 \equiv a_2 \mod n$ then for every $k \in \mathbb{N}$ we have $a_1^k \equiv a_2^k \mod n$.

(b) **(3pt)** Let $k \leq n$ be numbers in $\mathbb{N}$. Denote

$$C_k^n = \text{the set of all } k\text{-element subsets of } \{1, 2, \ldots, n\}$$

Prove by induction on $n \geq k$ that C_k^n has

$$\frac{n!}{k!(n-k)!}$$

elements.

Remark. You may use a similar approach as in the case of Problem 2 in Homework 4 assignment.

3. (5pt) Assume A, A', B, B' are sets, and $f : A \to A'$ and $g : B \to B'$ are bijections.

(a) **(1pt)** Prove that if $A \cap B = \varnothing = A' \cap B'$ then there exists a bijection $h : A \cup B \to A' \cup B'$.

(b) **(2pt)** Prove that there exists a bijection $h : A \times B \to A' \times B'$.

(c) **(2pt)** Prove that if there exists a bijection $h : A \to B$ then there exists a bijection $h' : A' \to B'$.