MATH 13 WINTER 2017 PRACTICE PROBLEMS

Below are some sample problems similar to those which will be given on the midterm. We may add more problems if there is interest or if we find some highly suitable ones.

1. Describe the following sets using separation/builder description/notation. When describing a given property, use only symbols $=, <, \leq$ and arithmetic operations like $+, -, \cdot$ and similar.

(a) The set $(-2, 5) \cup [1, 8) \cup \{0\}$ of real numbers.
(b) The set of all real numbers that lie between two consecutive integers.
(c) The set of all real numbers the square of which is not larger than 2.
(d) The set of all positive integers which can be expressed as a sum of two primes.
(e) The set of all rational numbers which can be expressed as a fraction with a positive power of 2 in the denominator.

2. Express the following statements in the symbolic language using symbols as instructed in Problem group 1 and then negate the statement.

(a) Every positive integer can be expressed as a sum of three squares of integers.
(b) For every real number r the equation $8x + r = 0$ has precisely one real solution.
(c) There are at least three different integers in the interval $(-2, 2)$.
(d) For every real number x larger than 1 the value $1/x$ is smaller than 1 but larger than 0.
(e) Between any two distinct rational numbers there is some rational number and some real number.

3. Prove the following statements. In each case choose a suitable type of proof.

(a) If a, b are odd integers then $a^2 - b^2$ is divisible by 4.
(b) If p is an odd integer then the equation $x^3 + x + p = 0$ does not have an integer solution.
(c) If a, b are integers divisible by an integer n then the product $a \cdot b$ is divisible by n^2.
(d) If a, b, c are three consecutive integers then the sum $a + b + c$ is divisible by 3 but the sum $a^2 + b^2 + c^2$ is never divisible by 3.
(e) The product of three consecutive integers is always divisible by 6.

3. Decide if each of the following statements is true or false. If true, prove it if false, disprove it. In each case choose a suitable type of proof. Here we consider sets $A, B, C \subseteq X$.

(a) If $A \subseteq B$ then $A \times B \subseteq B \times C$.
(b) If $A \subseteq B \cap C$ then $A \times A \subseteq B \times C$.
(c) $\mathcal{P}(A) \cap \mathcal{P}(A^c) = \varnothing$
(d) $\mathcal{P}(A) \cup \mathcal{P}(A^c) = \mathcal{P}(X)$.
(e) $\mathcal{P}(A) \setminus \mathcal{P}(B) \subseteq \mathcal{P}(A \setminus B)$.